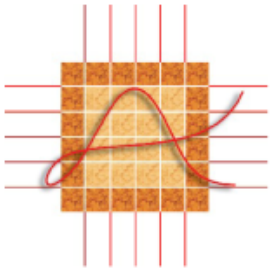

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Parallel Session
"*Variational Methods and Applications to Economics*"
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BOOK OF ABSTRACTS

Inverse Tensor Variational Inequalities & Market Models: the policymaker's point of view

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Extended abstract

During the last decades, variational inequalities on finite dimensional spaces have thoroughly been studied and their well-posedness theory has widely been developed. Such theory has various applications to applied science (e.g. transport planning, socio-economics phenomena, game theory) and it is only quite recently that the research has started to focus on its extension to a class of structured tensors, e.g. [2]. In particular, the topic of our interest will be inverse variational inequalities, that can be seen as a special case of general variational inequalities and can be considered to model various control problems. Classically, an inverse variational inequality problem formally consist in finding $x^* \in \mathbb{R}^n$ such that

$$f(x^*) \in \Omega : \quad \langle x - f(x^*), x^* \rangle \leq 0, \quad \forall x \in \Omega,$$

where Ω is a nonempty subset of $\mathbb{R}^n$ and $f : \mathbb{R}^n \to \mathbb{R}^n$. Unlike the classical variational inequalities, in such a problem the equilibrium state $f(x^*)$ has to belong to the feasible set, whereas the feasibility is not required for the variable x^*. It is only recently that the strict connection between classical variational inequalities and inverse variational inequalities has been unveiled, e.g. [4] and even in the case on finite dimensional space there are only a few available results in literature on the well-posedness theory for inverse variational inequalities, e.g. [3, 5].

Thus, our aim is to introduce the concept of Inverse Tensor Variational Inequality, to study their well-posedness theory, alongside with some applications to financial market modeling. Moreover, we are able to establish some characterizations of the well-posedness of a ITVI and we prove that under

suitable conditions, the well-posedness of a tensor inverse variational inequality is equivalent to the existence and uniqueness of its solution. Secondly, we show that the well-posedness of a tensor inverse variational inequality is equivalent to the well-posedness of an enlarged tensor variational inequality. Lastly, we analyze the policymaker's point of view for mathematically economic problems in which the equilibrium is expressed by a generalized Cournot-Nash principle and in particular we will focus on an oligopolistic market model. The results we present here are part of the joint work [1] with Annamaria Barbagallo and Serena Guarino Lo Bianco.

Parallel session: "Variational Methods and Applications to Economics", organized by Prof. Annamaria Barbagallo

Keywords
Inverse Tensor Variational Inequalities, market models, noncooperative agents

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General random time-dependent economic equilibrium model via stochastic variational inequalities

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Extended abstract [1]

In a Hilbert space setting, a random time-dependent oligopolistic market equilibrium problem in presence of both production and demand excesses is studied and the random time-dependent Cournot-Nash equilibrium principle by means of a stochastic variational inequality is characterized (see [1]). Then, some existence results to such problem are established and the stochastic continuity of the equilibrium solution is proved. Moreover a simple numerical example illustrates the theoretical results.

Keywords
Random time-dependent oligopolistic market equilibrium problem; Production and demand excesses; Stochastic variational inequalities.

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Algorithms for quasi-equilibria

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Extended abstract [1]

The quasi-equilibrium problem (QEP) is a quite natural generalization of the so-called abstract equilibrium problem (EP) where the constraints are given through a set-valued map describing how the feasible region changes together with the considered point. QEPs are modeled upon quasi-variational inequalities (QVIs) and generalized Nash equilibrium problems (GNEPs). As EP subsumes also other variational problems in a unique mathematical model, further "quasi" type models could be analysed through the QEP format beyond QVIs and GNEPs.

Unlikely QVI and GNEP, algorithms for the QEP format received limited attention. The goal of the talk is to discuss possible extensions of two classical algorithmic approaches for optimization and VIs to QEPs, i.e., fixed point and extragradient methods. The main difficulties arise from the moving feasible region: the iterates belong to different sets and any solution of QEP has to be a fixed point of the constraining set-valued map. Therefore, a range of convexity, monotonicity and Lipschitz assumptions both on the equilibrium bifunction and the constraining set-valued map must be met in suitable combinations to achieve convergence.

Keywords
Generalized Nash games, quasi-variational inequalities, fixed-point and extragradient techniques

[1] Special session: Variational Methods and Applications to Economics by Annamaria Barbagallo

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Quasivariational Inequalities for Dynamic Competitive Economic Equilibrium Problems in Discrete Time

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Extended abstract [1]

Equilibrium is a central concept in numerous disciplines including economics, management science, operations research, and engineering. We are concerned with an evolutionary quasivariational inequality which is connected discrete dynamic competitive economic equilibrium problem in terms of maximization of utility functions and of excess demand functions. We study the discrete equilibrium problem by means of a discrete time-dependent quasivariational inequality in the discrete space $\ell^2([0,T]_{\mathbb{Z}},\mathbb{R})$. We ensure an existence result of discrete time-dependent equilibrium solutions. Finally, We show the stability of equilibrium in a completely decentralized Walrasian general equilibrium economy in which prices are fully controlled by economic agents, with production and trade occurring out of equilibrium.

Keywords
Competitive economic; equilibrium, Discrete dynamic; Walras law; Variational inequality.

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Infinite dimensional tensor variational inequalities with applications to an economic equilibrium problem

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Extended abstract

In the last years, finite dimensional variational inequalities modeled in the class of tensors have been introduced and studied. Results on existence, uniqueness and regularity of solutions are available (see, for instance, [5], [1], [2] and the reference therein). Such theory has various applications to applied science (e.g. transport planning, socio-economics phenomena, game theory) and it is only quite recently that the research has started.

We present a general oligopolistic market equilibrium model in which each firm produces several commodities and evolves in time.

To this end, tensor variational inequalities in Hilbert spaces are introduced and a special case of these, the so-called evolutionary tensor variational inequalities, are analyzed. For such inequalities, some existence results and a Minty-Browder type characterization are obtained. Moreover, other useful properties, like the point-to-point formulation for evolutionary tensor variational inequalities and some theorems of continuity with respect to the time variable, are established.

The dynamic general oligopolistic market equilibrium problem is the problem of finding a trade equilibrium in a supply-demand market between a finite number of spatially separated firms which produce several different goods in a fixed time interval and act in a noncooperative behavior. This model is the time-dependent version of the economic equilibrium problem presented in [1] and extensively studied in [2] where the ill-posedness and the stability analysis are studied. The motivation of such an introduction derives by the relevance of the time in equilibrium models, as M.J. Beckmann and J.P. Wallace underlined in [4]: "the time-dependent formulation

of equilibrium problems allows one to explore the dynamics of adjustment processes in which a delay on time response is operating".

The results we present here are part of the joint work [3] with Annamaria Barbagallo.

Parallel session: "Variational Methods and Applications to Economics", organized by Prof. Annamaria Barbagallo

Keywords
Tensor variational inequality, existence and regularity results, time interval

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Addressing monotone nested variational inequalities in multi-portfolio optimization contexts

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Extended abstract [1]

We consider nested variational inequalities arising in multi-portfolio optimization, whenever trades from different accounts are pooled for common execution. We propose a projected averaging Tikhonov-like algorithm to treat these problems numerically: convergence of the method is proven under, as far as we are aware, the weakest conditions among the ones resorted to in the literature.

Keywords
Hierarchical programs; Variational Inequalities; Converging numerical procedures; Multi-portfolio selection

[1] Special session: [Variational Methods and Applications to Economics].
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Evaluating the effects of environmental regulations on the European pulp and paper industry

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Extended abstract[1]

This paper proposes an optimization problem for a pulp and paper supply chain in the context of circular economy. The developed supply chain model accounts for forest deployment, production of paper using both virgin and recycled pulp, the collection of used paper and its re-utilization. The aim of

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the paper is to investigate the sustainability of this sector and to analyze the impacts of carbon emission regulations on the amount of the paper produced and recycled. The case study is focused to the European pulp and paper industry that is subject to the European Union Emission Trading Scheme.

Keywords
Optimization; pulp and paper supply chain network; environmental regulations; circular economy.

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A network centrality measure based on the equilibrium social welfare in network games

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Extended abstract[1]

A network game is a noncooperative game modeling the social and economic interactions among players through a graph. Each node represents a player which can interact only with its neighbors in the graph, while possible congestion effects are due to all the network players. In this talk, we consider a class of network games with bounded strategy sets and quadratic utility functions describing local complementarities and possible global congestion. First, we provide a representation formula, based on the Katz-Bonacich centrality measure [**?**], for the Nash equilibrium in the case where some of its components lie on the boundary. Moreover, we propose a new network centrality measure to analyze the importance of a player by measuring the percentage variation of the social welfare computed at the Nash equilibrium when the player is removed from the network. Notice that the proposed centrality measure can takes on negative values if the social welfare of the network increases after removing a player (similar situation to the well known Braess paradox). Finally, we compare the proposed measure with well known topological centrality measures and some recent Nash equilibrium-based centrality measures [**?**].

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Keywords
Network game; Nash equilibrium; social welfare; network centrality measure.

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