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STOCHASTIC METHODS IN FINANCE AND INSURANCE

BOOK OF ABSTRACTS

Organisers:

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What are Asset Price Bubbles?
A Survey on Definitions of Financial Bubbles

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Extended abstract [1]

Financial market bubbles and crashes have caused large economic damages, e.g., the 2008 financial crisis but also the Black Thursday in 1929. However, precise definitions of what are asset price bubbles are difficult and ambiguous across different application areas.

We contribute by providing a systematic overview of definitions of asset price bubbles sorted by application areas and evaluate the real-time applicability of the definitions.

A main distinction is made between definitions that use fundamental values and definitions that refer to price changes over time [1]. Since many economists prefer to define bubble prices as deviations from fundamental values, the specification of fundamental value is needed:

First, the fundamental value can be taken as the sum of all discounted expected future dividends (up to a resale time plus the expected resale value) or retrospectively the sum of realized discounted dividends. Or as the total assets (balance sheet or resale value) divided by the number of shares. Note that traders do not have homogeneous believes about future dividend payments (cf. [2]).

Second, expected future cash flows do not necessary match with the actually paid dividends when looking back at historical data (cf. [3]). Third,

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there are assets that do not pay monetary dividends. Moreover, there is the question of whether money/gold/cryptocurrencies are bubbles [4]. Fourth, if a price is above its fundamental value, this does not mean that it will necessarily fall. But there is also no reason for it not to do so (cf. [2]). This idea is followed up in stochastic analysis [5]. If one defines prices as stochastic processes, it is no longer just a price path that is a bubble, but the whole process. That is, it is assumed that the "bubble property" is inherent in the whole dynamic, whether or not "bubble dynamics" are actually observed in a realization of the process.

Keywords
asset price bubble; fad; financial crisis; local martingale; fundamental analysis.

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A Stochastic Control Approach to Public Debt Management

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Extended abstract [1]

Public debt management is one of the most relevant topics in Economics, especially after economic crises due to wars, pandemics or economic recession. We discuss a class of debt management problems in a stochastic environment model. We propose a model for the Debt-to-GDP ratio where the government interventions (via fiscal policies) affect the public debt and the GDP growth rate at the same time. We allow for a stochastic interest rate on debt and possible correlation with the GDP growth rate. Indeed, both the interest rate and the GDP growth depend on a stochastic factor, which may represent any relevant macroeconomic variable, such as economic conditions. Moreover, shocks on debt and GDP can be correlated. We tackle the problem of a government whose goal is to determine the fiscal policy (quantity of surplus or deficit) in order to minimize a general functional cost. We prove that the value function is a viscosity solution to the Hamilton-Jacobi-Bellman equation and provide a Verification Theorem based on classical solutions. We investigate the form of the candidate optimal fiscal policy in many cases of interest, providing interesting policy insights. Finally, we discuss two applications to the debt reduction problem and debt smoothing, providing

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explicit expressions of the value function and the optimal policy in some special cases.

Keywords
Optimal stochastic control; government debt management; optimal fiscal policy; Hamilton-Jacobi-Bellman equation.

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Forecasting Portfolio Returns with Skew-Geometric Brownian Motions

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August 22, 2021

Extended abstract [1]

The goal of this work is to find a minimum tracking error portfolio that can not only be adopted as an automated alternative to ETFs but also potentially be used to anticipate market changes in the target index. This has been achieved by following the existing literature, such as Mahoney [2], where the idea is to model the cross dependency between assets and, thus, to minimize the tracking error of a portfolio.
However, it is well known that returns of financial assets are not normally distributed but they rather follow a t-skew distribution (see [4]) as scale mixtures of skew-normal distributions. For this reason, given an index $\mathcal{I}$ and n its sub-indices $(S_i)_{i\in[1,n]}$, we assume that their (log-)returns follow skew-geometric Brownian motions which are correlated among themselves.

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Given a filtered probability space $(\Omega, \mathbb{P}, \mathcal{F}, (\mathcal{F}(t))_{t\geq 0})$, a skew-geometric Brownian motion S is defined by

$$S(t) = S(0)\,exp\Big(\mu(t)t + \sigma(t)Y(t)\Big), \tag{1}$$

where $\mu, \sigma \in \mathbb{R}_+$, $Y(t) = X_t - \mathbb{E}[X(t)]$, and $X(t)$ is such that

i) $X(0) = 0$;

ii) for any $t \geq 0$, $X(t)$ has continuous sample paths;

iii) for any $t \geq 0$, $X(t) \sim SN(0, t, \beta)$.

X is called skew-Brownian motion and Y is its centred version. As observed in [1, Proposition 2.1], a natural construction of a skew-Brownian motion consists of the sum of a Brownian motion and a reflected Brownian motion, i.e.,

$$X(t) = \sqrt{1-\delta^2}\cdot W_1(t) + \delta|W_2(t)|, \tag{2}$$

where $\delta = \frac{\beta}{\sqrt{1+\beta^2}}$, and W_1 and W_2 are independent Brownian motions. In particular, the following properties hold true

i) $X(t)$ is a $\mathbb{P}$-submartingale;

ii) For any $0 \leq s < t$

$$\mathbb{E}[Y(t)|\mathcal{F}(s)] = u_1(s) - u_2(s) + 2u_2(s)\Phi\left(\frac{u_2(s)}{\delta\sqrt{t-s}}\right) + \delta\sqrt{\frac{2(t-s)}{\pi}}\cdot\phi\left(\frac{u_2(s)}{\delta\sqrt{t-s}}\right) - \delta\sqrt{\frac{2t}{\pi}}, \tag{3}$$

and

$$Var(Y(t)|\mathcal{F}(s)) = (t-s) + u_2^2(s) - \left(u_2(s)\left(2\Phi\left(\frac{u_2(s)}{\delta\sqrt{t-s}}\right) - 1\right) + \delta\sqrt{\frac{2(t-s)}{\pi}}\cdot\varphi\left(\frac{u_2(s)}{\delta\sqrt{t-s}}\right)\right)^2, \tag{4}$$

where φ, Φ are the (standard) normal PDF and CDF, $u_1(s), u_2(s)$ are realizations of $\sqrt{1-\delta^2}\cdot W_1(s)$ and $\delta|W_2(s)|$, respectively.

For any $t \geq 0$ and $i \in [1, n]$, the index and the assets are defined by the following dynamics

$$\begin{cases} \mathcal{I}(t) = \mathcal{I}(0)\,exp\big(\mu_0(t)t + \sigma_0(t)Y_0(t)\big) \\ S_i(t) = S_i(0)\,exp\big(\mu_i(t)t + \sigma_i(t)Y_i(t)\big), \end{cases} \tag{5}$$

where Y_0 is a centred skew-Brownian motion with shape parameter β_0, defined as

$$Y_0(t) = \sqrt{1-\delta_0^2(t)} \cdot W_{1,0}(t) + \delta_0(t)|W_{2,0}(t)| - \delta_0(t)\sqrt{\frac{2t}{\pi}}, \tag{6}$$

with $W_{1,0}$, $W_{2,0}$ independent Brownian motions, $\delta_0(t) = \frac{\beta_0(t)}{\sqrt{1+\beta_0^2(t)}}$ and we let

$$Y_i(t) = \rho_i(t)Y_0(t) + \sqrt{\left(1-\frac{2\delta_0^2(t)}{\pi}\right)(1-\rho_i^2(t))} \cdot W_i(t), \tag{7}$$

where W_i is a Brownian motion independent of Y_0, and $\rho_i \in (-1, 1)$.
Our portfolio replicates the index $\mathcal{I}$, at a fixed time $s > 0$, if the (squared) tracking error

$$TE^2(s) = Var\left(R_0(s) - \sum_{i=1}^{n} w_i(t)R_i(t)\right) \tag{8}$$

is minimum. More specifically, one has to solve the following problem

$$\begin{cases} \min_{\boldsymbol{w}} Var\left(R_0(s) - \sum_{i=1}^{n} w_i(t)R_i(t)\right), \\ \textit{subject to:} \begin{cases} \sum_{i=1}^{n} w_i(s)\mathbb{E}[R_i(s)] = R_0(s) \\ \sum_{i=1}^{n} w_i(s) = 1; \end{cases} \end{cases} \tag{9}$$

where $R_0(t) = \ln\left(\frac{\mathcal{I}(t)}{\mathcal{I}(0)}\right)$ and $R_i(t) = \ln\left(\frac{S_i(t)}{S_i(0)}\right)$ denote the log-returns. Once determined $\widehat{\boldsymbol{w}}$, the future index return can be predicted by

$$R_0^F(t) = \sum_{i=1}^{n} \widehat{w}_i(s)\mathbb{E}[R_i(t)|\mathcal{F}(s)], \tag{10}$$

for any $t > s$.

In order to generalize problem (9), and due to the difficulty of correlating $n > 2$ skew-geometric Brownian motions, we built n portfolios where, in the j-th portfolio, we assume that only the j-th subindex is correlated with the others, for any $j \leq n$. This solution is new in the literature and allows us to replicate the index with its n sub-indices while minimizing the tracking error.
Fixed $t \geq 0$ and $j \in [1, n]$, we name $\mathcal{R}^j$ the returns at time t of the portfolio

consisting of $S_1, S_2, ..., S_n$, in which S_j is correlated both with $\mathcal{I}$ and S_i, with $i \in [1, n]$. We can write

$$\mathcal{R}^j(t) = w_j(t)R_j(t) + \sum_{\substack{i=1 \\ i \neq j}}^{n} w_i(t)R_{j,i}(t), \tag{11}$$

where

$$R_{j,i}(t) = \mu_{j,i}(t)t + \sigma_{j,i}(t)Y_{j,i}(t),$$
$$Y_{j,i}(t) = \rho_{j,i}Y_j(t) + \sqrt{\left(1 - \frac{2\delta_j^2}{\pi}\right)(1 - \rho_{j,i}^2)} \cdot W_{j,i}(t) \tag{12}$$

and

$$Y_j(t) = \sqrt{1 - \delta_j^2(t)} \cdot W_{1,j}(t) + \delta_j(t)|W_{2,j}(t)| - \delta_j(t)\sqrt{\frac{2t}{\pi}}, \tag{13}$$

with $W_{1,j}, W_{2,j}, W_i$ independent Brownian motions. To replicate the index returns, and so, to predict R_0 (for any t), we need determinate appropriate weights γ_j for each $\mathcal{R}^j$, so that their linear combination reproduces the future value of R_0 with the minimum risk. As a measure of risk, we take the square of the traking error (for any fixed time $s > 0$), i.e.,

$$TE^2(s) = Var\left(R_0(s) - \sum_{j=1}^{n} \gamma_j(s)\mathcal{R}^j(s)\right), \tag{14}$$

that can be rewritten as

$$Var(R_0(s)) + \sum_{j=1}^{n} \gamma_j^2(s)Var(\mathcal{R}^j(s)),$$

due to the indipendence of the variables R_0 and $\mathcal{R}^j$, for any $j \in [1, n]$.
In other terms, fixed $s > 0$, we have to solve the following optimization problem

$$\begin{cases} \min\limits_{\boldsymbol{\gamma}} Var(R_0(s)) + \sum_{j=1}^{n} \gamma_j^2(s)Var(\mathcal{R}^j(s)), \\ \textit{subject to:} \begin{cases} \sum_{j=1}^{n} \gamma_j(s)\mathbb{E}[\mathcal{R}^j(s)] = R_0(s) \\ \sum_{j=1}^{n} \gamma_j(s) = 1. \end{cases} \end{cases} \tag{15}$$

Once determined $\widehat{\boldsymbol{\gamma}}$, the future index return can be predicted by

$$R_0^F(t) = \sum_{j=1}^{n} \widehat{\gamma}_j(s)\mathbb{E}[\mathcal{R}^j(t)|\mathcal{F}(s)], \tag{16}$$

for any $t > s$.

Observe that the sum of (independent) skew-normal variables (involved in Eq. (8)-(10) and (14)-(16)) is not skew-normal distributed. The exact distribution, named $\mathcal{D}$, is given by [3, Theorem 2.1]. This makes the computation of the (conditioning) expected values appearing in Eq. (10)-(16) not trivial. The explicit computation of such expectations is one of the main results of this work.

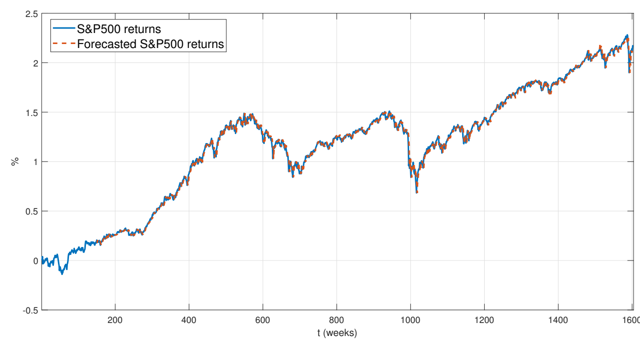
We test the above procedures (jointly with a suitable calibration) on the S&P 500 (representing developed markets) and Bovespa (representing emerging markets) returns. Numerical results show that the proposed solutions replicate the index with a much smaller tracking error than that of the ETFs considered (see Table 1 and Figures 1, 2). For comparison, in Tsalikis et al. (2019) [5] the TE^2 reported for studied ETFs is 0.0196 with a minimum of 0.0026 and a maximum of 0.0946.

Table 1: Results of forecasting as obtained with the optimization problem in Eq. (9) (not correlated assets) or that one specified in Eq. (15) (correlated assets)

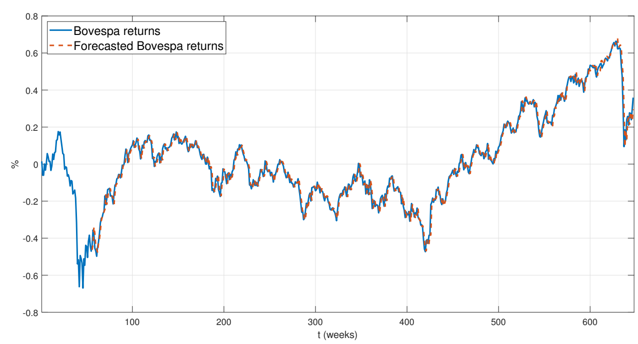
Index	**Opt. Problem**	**RMSE**	**MAPE**	**average TE^2**
S&P500	(9) (15)	0.1351 0.0325	0.1061 0.0225	$1.2379 \cdot 10^{-4}$ $1.3514 \cdot 10^{-5}$
BOVESPA	(9) (15)	0.2533 0.0326	0.1989 0.0256	$2.5023 \cdot 10^{-4}$ $3.2179 \cdot 10^{-5}$

Keywords

Forecasting, Portfolio optimization, Skew-normal Brownian motions.



(a) S&P500 returns forecast from 1991 to 2020 (1452 weeks).



(b) Bovespa returns forecast from 2008 to 2020 (592 weeks).

Figure 1: Forecasted returns (through Eq. (16)).

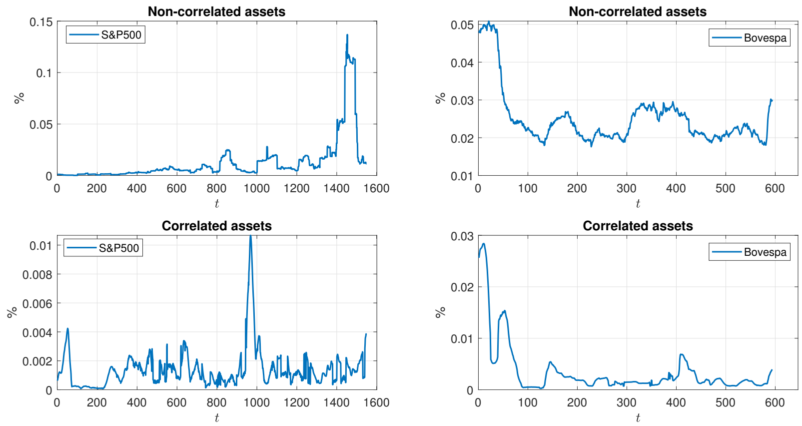


Figure 2: Squared tracking error (TE^2) computed both for the non-correlated and correlated case (see Eq. (8) and (14)), with respect to the S&P500 and Bovespa returns, for any time t.

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On a class of partially observed systems arising in singular optimal control

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Extended abstract [1]

Partially observed systems model phenomena that appear in various disciplines, such as engineering, economics, and finance, where some quantity of interest, described by a stochastic process called signal, is not directly measurable or observable. The signal process affects another quantity, the observed process, through which one can obtain probabilistic estimates of the state of the unobserved signal. The estimate that one seeks is provided by the filtering process, defined as the conditional distribution of the signal at each time $t \geq 0$, given the observation available at time t.

This estimate is required, for instance, in optimal control problems with partial observation, where an agent (or controller) aims at optimizing some functional, depending on the stochastic processes previously introduced, by means of a control process. In continuous time, these problems have been deeply studied in the literature. However, to the best of our knowledge, a particularly relevant case for applications has not yet received proper attention: the singular control case. Indeed, few papers study singular control problems for partially observed systems and they do so only (excepting [4], where linear dynamics are considered) in the case where the control process acts on the observation (see, e.g., [1, 2, 3, 5]). Instead, the case where the

[1]

Speaker: Alessandro Calvia, acalvia@luiss.it control acts on the signal process is more delicate, from a technical point of view, and requires a careful novel analysis.

In this talk, we will introduce a class of singular control problems with partial observation, discuss a motivating application to a pollution control problem, and provide the explicit filtering equation (i.e., the SPDE satisfied by the filtering process), together with a uniqueness result. These results lay the ground to solve the corresponding singular optimal control problem under partial observation.

Parallel session: Stochastic Methods in Finance and Insurance.
Organizers: Katia Colaneri, Alessandra Cretarola.

Keywords
Stochastic filtering; singularly controlled systems; reference probability measure; Zakai equation; Kushner-Stratonovich equation.

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Maximum risk diversification for portfolio selection

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Extended abstract [1]

In this paper we address the problem of finding portfolios with maximum diversification, possibly with the addition of return constraints. The diversification measure is based on a convexity ratio between the risk of a convex combination of assets and the convex combination of their risks.
Our contribution is manifold: we extend the maximum diversification approach for volatility to general subadditive and positive homogeneous risk measures; we create a bridge between Risk Parity [5] and the most diversified portfolios of [4]; we add a target return to the maximum diversification approaches in the Gain-Risk analysis style. Finally, we provide an extensive empirical analysis based on seven real-world datasets, highlighting encouraging out-of-sample performances of our approach compared to the classical ones.
More precisely, for a subadditive and positive homogeneous risk measure ϱ, we define the Maximum Diversification (MD) portfolio as the one maximizing the following Diversification Ratio:

$$DR(x) = \frac{\sum_{i=1}^{n} x_i \varrho(R_i)}{\varrho(\sum_{i=1}^{n} x_i R_i)},$$

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where x_i is the percentage of capital invested in the asset i and R_i is the linear return of the asset i. By construction, $DR(x) \geq 1$.

The optimization problem defining the MD portfolio is a convex fractional programming problem that can be reformulated as a convex problem. We also add a return constraint to the Maximum Diversification problems, to obtain return-diversification frontiers in the same spirit as the classical return-risk efficient frontier. We provide the explicit formulations for the Maximum Diversification approach for some specific risk measures typically used in asset allocation and we show that the MD problem admits LP formulations in the case of Mean Absolute Deviation and CVaR risk measures. We apply all the proposed portfolio strategies on five real-world datasets, which have been used in other empirical analyses on portfolio selection (see, e.g., [1, 3, 2]), and on two new up-to-date datasets.

Keywords
Risk Diversification, Portfolio Selection, Convex Risk Measures, Subadditivity.

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Some probability distortion functions in behavioral portfolio selection

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Extended abstract [1]

In this paper we discuss some behavioral portfolio selection models as introduced by [1] based on Cumulative Prospect Theory ([2] and [4]). In particular, we consider two different probability distortion functions, that is strictly increasing functions mapping the probability interval $[0, 1]$ into $[0, 1]$, respectively advanced by Tversky and Kahneman [4] and by Prelec [3]. The resulting mathematical programming problem turns out to be highly non-linear and non-differentiable, so it cannot be solved applying traditional optimization techniques. For these reasons, according to what already done in [1], we adopt a solution approach based on the metaheuristic Particle Swarm Optimization (PSO). PSO is an iterative bio-inspired population-based metaheuristic for the solution of global unconstrained optimization problems. Given that our optimization problems are global constrained, we reformulate them into unconstrained ones using a nondifferentiable penalty function method already applied in the financial context. Lastly, we apply the developed behavioral portfolio selection models to the European equity market as represented by the STOXX Europe 600 Index and compare their performances.

Keywords
Behavioral finance; Cumulative Prospect Theory; portfolio selection; probability distortion function; Particle Swarm Optimization (PSO).

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A stochastic model for capital requirement assessment for mortality and longevity risk, focusing on idiosyncratic and trend components

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Extended abstract [1]

This paper provides a stochastic model, consistent with Solvency II and the Delegated Regulation (see [3]), to quantify the capital requirement for demographic risk. In particular, we present a framework that models idiosyncratic and trend risks exploiting a risk theory approach in which results are obtained analytically. The first risk, which results are obtained in closed formulas, is compared with the analogous quantity analysed in a Local GAAP context (see [4]). With reference to trend risk, on the other hand, the model was easily adapted to the stochastic framework presented by Brouhns et al. (see [2]).
We apply the model to non-participating policies and quantify the Solvency Capital Requirement for the aforementioned risks in different time horizons.

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Keywords
Life insurance; Mortality & longevity risk; Risk theory; Solvency II; Solvency Capital Requirement.

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How does correlation impact Value-at-Risk bounds?

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Extended abstract [1]

Given a portfolio of n risks $X_1, ..., X_n$, determining the Value-at-Risk (VaR) of their sum $S = \sum_{i=1}^{n} X_i$ is a central problem in insurance and finance. Some aggregation formulas that are used in the insurance and financial industry implicitly assume that the knowledge of marginal distributions and of some measure of dependence (e.g., the average correlation) leads to an appropriate estimation of the VaR. We challenge this idea by investigating under which conditions the unconstrained VaR bounds (which are the maximum and minimum VaR for $S = \sum_{i=1}^{n} X_i$ when only the knowledge on the marginal distributions of the components X_i is assumed) coincide with the VaR bounds when in addition one has information on some measure of dependence. For the sum of $n = 2$ risks the problem of finding the unconstrained VaR upper bound has been completely solved (see e.g. [1]). We show that having information on a measure of association such as Pearson

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correlation, Spearman's rho or Kendall's tau has typically no impact on the VaR upper bound, whereas it can improve the lowest possible VaR. We also study the general case of n risks using the Range Value-at-Risk (RVaR) as risk measure (which is a generalization of VaR). Specifically, we derive RVaR bounds under a constraint on the average correlation and provide sufficient conditions for their sharpness. Corresponding VaR and TVaR bounds are obtained as special cases.

A main insight is that for the probability levels usually considered in Risk Management and capital allocations (e.g., $p = 95\%$ or higher) it is quite difficult that a constraint involving one of the dependence measures under consideration actually leads to a reduction of a risk measure worst-case scenario.

Keywords
Risk bounds; Value-at-Risk; Pearson correlation; Spearman's rho.

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A discrete-time Markov chain for estimating new entrants into professional pension funds

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Extended abstract

I propose a discrete-time Markov chain for estimating new entrants into professional pension funds, highlighting the interactions between demographic, socio-economic and regulatory variables. The model considers the effects of trends in population, education choices, and the appeal of the profession. It can help to improve the quantitative analysis of pension schemes by addressing the demographic risk related to the uncertain number of future new contributors. This risk is particularly acute for pension schemes that admit a homogeneous class of people (such as professional society members, academically qualified workers, etcetera). Indeed, changes in the job market can have a relevant influence on the number of contributors, thus affecting the sustainability of these schemes.

The intuition behind the model is that, in the medium term, trends in academic education can anticipate changes in the job market and preferences for highly skilled professions. Similarly, fertility trends can anticipate the number of future young adults, thus influencing the overall occupational structure of employment in the long term. In this perspective, the study addresses the issue with a mathematical formalization of the problem and an application to the national pension fund of Italian Chartered Accountants (CNPADC).

Keywords
Demographic risks; Pension funds; New entrants; Markov chain; CNPADC.

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The Complex Step Method for Financial Greeks

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Extended abstract [1]

The complex step approximation (CSA) is a well known numerical method for the calculus of first order derivatives of highly non linear functions, see for instance [1] and references therein .

[2] presents the application of the CSA method for the calculus of the Greeks of barrier options, using analytical formulas in the Black and Scholes framework. However, to te best of our knowledge, a complete discussion of CSA method to the calculus of financial Greeks is not presented in literature. This work aims to fill this gap.

We analyse both theoretically and empirically the CSA performance associated with pricing methods as the Fourier transform and Monte Carlo simulations. Moreover, we compare the CSA method with the other most widespread methods for the calculus of financial Greeks, i.e. the finite difference, the forward and the adjoint algorithmic differentiation. Furthermore, a section of the work is dedicated to discuss the simulation techniques of some processes, as for instance the Heston model, when the relative stochastic differential equation is complex valued. Finally, we discuss some extensions of the CSA method for the calculus of second order Greeks.

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Keywords
complex step method, financial greeks.
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Surrender and path-dependent guarantees in variable annuities: integral equation solutions

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Extended abstract [1]

Introduced in the US insurance market in the 1970s, variable annuities (VAs) have spread in insurance markets at the beginning of 1990s when insurers have begun to embed in such policies additional guarantees, thus making them more flexible than traditional life insurance contracts. Currently, VAs represents the most traded retirement income products in Europe, Japan, and particularly in US where, according to the preliminary results from the Secure Retirement Institute (SRI) US Individual Annuity Sales Survey, in the first quarter 2021 total VA sales were $29.9 billion in the first quarter, up 15% from prior year.

The aspects evidenced above motivate academics and practitioners to develop accurate and efficient evaluation methods for VAs equipped with a large variety of possible guarantees ranging from accumulation, annuity or death benefits to income or withdrawal benefits, as well as tax incentives.

In this paper, we concentrate the attention on VA policies providing the policy-holder the opportunity to lapse the contract before maturity by exercising a surrender option on the embedded guarantee. We provide evaluation methods for three different models:

- **Model 1**: VA with a guaranteed minimum maturity benefits (GMMB) characterized by constant guarantees, representing the basic case;
- **Model 2**: VA with a GMMB characterized by constant guarantee with an up-and-out barrier;

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- **Model 3**: VA with a GMMB characterized by lookback path-dependent guarantees.

Working under the settings presented in **Model 1**, **Model 2**, and **Model 3**, we start by observing that Shen et al. [1], and Jeon and Kwak [2],[3] have already proposed Volterra integral equation frameworks. It is worth evidencing that these kinds of equations are complex tao be numerically solved and standard approaches, such as tree methodologies or recursive integration methods, require a large number of steps to achieve accurate approximations. We observe that, applying the theoretical findings of Peskir and Shiryaev [4], the evaluation of the guarantees embedded in VA contracts may be conducted to a free boundary problem for which a non-standard Volterra integral equation solution can be derived. In particular, the integrals appearing in the non-standard Volterra integral equations associated to the VA evaluation problems characterizing **Model 1**, **Model 2**, and **Model 3**, have specific properties allowing us to develop a new flexible numerical method based on the Mean-Value Theorem combined with some interpolation procedures. Indeed, each integral contains both a function satisfying the Lipschitz condition and a bounded function, so that we are able to establish a non-linear system of equations and to solve it numerically in order to find an accurate approximation of the early exercise boundary. We assess the method accuracy in comparisons with a binomial tree.

Keywords
variable annuity; surrender option; lookback path-dependent guarantee; Volterra integral equation; lattice model.

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A performance analysis of portfolio insurance strategies with guaranteed minimum equity exposure

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Extended abstract [1]

A Portfolio Insurance (PI) strategy represents a route to protect the invested capital, based on the comparison between the terminal value of the portfolio and a predetermined threshold [3, 4]. In this work we focus on option-based strategies (OBPI), where the total amount to be invested is conveniently split in a risk-free security and a European option. However, such a type of strategy can guarantee neither a protection of the initial capital in case of sudden underlying price drawdowns, nor even to prevent the allocation of resources in the risky component from vanishing, the so called cash-in risk. To overcome the former key-point, the Constant Proportion-Portfolio Insurance (CPPI) dynamic strategy was first introduced and progressively variously modified [1]. To avoid the latter, a guaranteed minimum equity exposure (GMEE) was introduced in the invested capital shares for the risky securities: to reap the benefits from using both dynamic and static strategies, a CPPI-OBPI mixed methodology can be introduced, in which the risky asset is given by a CPPI portfolio with GMEE [2]. We investigate the role played by GMEE in PI strategies under a Vasicek-Heston model, as a vehicle to test the efficiency of such methodologies. Due to the lack of a two-sided market

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for options linked to PI strategies, the underlying model must be calibrated under P. We choose the SPX time series, and we apply the Wang transform [5] to artificially reconstruct the martingale condition of the process. We measure specific performance indicators and draft the corresponding risk profiles by considering different market scenarios and allowing for transaction costs. The performance analysis confirms the ability of the mixed strategy both to guarantee the equity market participation and to avoid the cash-in risk, regardless of market conditions. Moreover, we show that the presence of GMEE represents a further form of protection, as it ensures lower expected losses than the other strategies.

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Keywords
Portfolio insurance; CPPI; OBPI; Guaranteed minimum equity exposure; Risk-adjusted performance.

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Informed trading, short-sale constraints, and leverage effect in equity returns

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Extended abstract [1]

I model informed trading subject to short-sale constraints and find that short-sale constraints can cause the asymmetric volatility effect (also known as the "leverage effect"). I offer an infinite-horizon model with overlapping generations of private information and stationary time series of returns. This model builds up on the framework of Kyle model [1] augmented with a short-sale constraint. My model features higher maker opacity in the situation in which informed agents have negative signals because with the short-sale constraints, they reveal less information to the other participants. I consider a collection of settings and find that the magnitude of the leverage effect is driven by the assumptions of the probability distribution of the asset's fundamental value. Additionally I find that fat-tailed fundamental values can generate a persistent volatility effect irrespective of the short-sale constraints.

In my model the uninformed market participants make Bayesian updating based on the publicly available information, and the informed traders make their trading decisions so as to maximize their expected profits. Due to the constraints, the model lacks analytical solution. I use numerical techniques to solve it. The fact that my model predicts both asymmetric volatility and persistent volatility, makes it parallel to the branch of econometric literature that studies both phenomena in junction, e.g [2], [3].

Keywords
Leverage effect, Volatility, Informed trading, Market microstructure, Short-sale constraints, Skewness, Equities, Price efficiency, Uncertainty.

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XVA approximation for European claims: a BSDE approach

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Extended abstract [1]

We consider the problem of computing the Value Adjustment of European contingent claims when default of either party is considered, possibly including also funding and collateralization requirements. As shown in Brigo et al. in a series of papers (see [1] and the references therein), this leads to a more articulate variety of Value Adjustments (the XVA's) for the claim price that introduce some nonlinear features. The adjusted price can be characterized as the solution to a Backward Stochastic Differential Equation (BSDE). When exploiting a reduced-form approach for the default times, the expectation representing the solution of the BSDE is usually quite hard to compute even in a Markovian setting, and one might resort either to the discretization of the Partial Differential Equation characterizing it or to Monte Carlo Simulations. Both choices are computationally very expensive, in particular when considering stochastic default intensities. In this paper we suggest viewing such an expectation as a smooth function of the correlation parameters and to approximate it by its Taylor polynomial expansion around the zero vector (the independent case), in the hope that the first or second-order are enough to provide an accurate approximation. We apply our method to estimate the price contribution that comes from considering stochastic default intensities correlated with the underlying's price. We remark, though, that we can straightforward extend the same technique to include further stochastic factors. In order to evaluate Taylor polynomial's coefficients, we follow a two-step procedure to exploit, whenever possible, explicit formulae from option and bond's pricing theory. First, we condition the underlying's price with respect to the Brownian motions driving the intensities, retrieving a conditional explicit formula. Then, assuming the intensities to be described by affine models, we represent the single terms of the expansion using a change of Numéraire technique to

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disentangle the correlations between the asset's price and the default intensities. The affinity of the processes makes it possible to use a "bond-like" expression for the default component. The numerical discussion at the end of this work shows that, at least in the case of the Cox-Ingersoll-Ross (CIR) intensity model, even the simple first-order approximation is accurate compared to the benchmark Monte Carlo evaluation.

Keywords
Pricing; Credit Value Adjustment; Defaultable Claims; XVA; Affine Processes.

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Optimal Firm's Dividend and Capital Structure for Mean Reverting Performance

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Extended abstract [1]

Since the seminal paper by [1], the firms' dividend policy is acknowledged to be rather stable over time. However, the theoretical motives behind this empirical observation are still being questioned. In general, theoretical explanations for dividend smoothness derive from informational asymmetries and, in particular, from the signaling content of dividends (see, for instance, [3]). Managers are reluctant to cut on dividends, because this would be interpreted as an indication of a drop in profitability expectations. Interestingly, few theoretical papers have analysed dividend smoothing when information is symmetric. More in general, very few papers have tried to model the firm's optimal dividend and capital structure decisions jointly. In this paper, we combine these two features and explore the optimal dynamic firm dividend and leverage problem, with neither asymmetric information nor frictions. In such a framework, we are able to obtain both dividend smoothness and leverage stability.

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We consider a stand-alone firm, with no separation of ownership and control. The owner/manager of the firm obtains inter-temporal (constant relative risk aversion) utility from dividends payouts over a given finite time horizon and terminal shareholder value. We solve the problem of maximizing the expected present value of such utility, while computing both the optimal dividend and the optimal firm's debt.

Our setup, which is partly based on [2], considers a system driven by two stochastic state variables: the value of the firm's equity, and the return on firm's assets (ROA). These two variables are assumed to be correlated and ROA is modeled as a mean reverting process, following the empirical evidence suggests. We obtain a quasi-explicit solution for both the optimal dividend and debt and characterize it in terms of the model parameters. We run numerical simulations by calibrating our model to U.S. firm data. Our analysis highlights that even when time horizon is relatively short, the mean-reversion property of the stochastic profitability leads to an almost deterministic dividend yield (i.e. the ratio between dividends and equity). Leverage is much more volatile since its adjustments respond to the short-term fluctuations of current profitability. Through a parameter sensitivity analysis, we find that when profitability is not mean reverting, dividend smoothness breaks and volatility increases across paths both in dividends and leverage. As a by-product, the numerical simulations show that our model is able to reproduce the empirically observed negative relation between profits and leverage.

Keywords
dividend policy, capital structure, profit mean-reversion, closed-form, stochastic optimization.

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Optimal investment and proportional reinsurance in a Markov modulated market model under forward preferences

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Extended abstract [1]

We study the optimal investment and proportional reinsurance problem of an insurance company whose investment preferences are described via a dynamic forward utility of exponential type. Optimal reinsurance and investment problems in the actuarial framework have been widely investigated [see e.g. [2], [1] and references therein]; but, to the best of our knowledge, have only been addressed under classical backward preferences. According to Musiela and Zariphopoulou [[3], [4]], we provide the definition of a forward dynamic performance process. Specifically, given the initial utility function as input parameter, the forward dynamic utility for an arbitrary upcoming investment horizon is specified by means of the solution to a suitable stochastic control problem, such that the supermartingale property holds for any

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admissible strategy, and the martingale property holds along the optimal strategy. We consider a stochastic factor model where insurance and financial markets are mutually dependent via the presence of a Markov process affecting the claim arrival intensity as well as asset price dynamics. We construct the value function and we prove that it provides a forward dynamic utility. Then, we characterize the optimal investment strategy and the optimal retention level. We also present numerical experiments and provide a sensitivity analysis with respect to the parameters of the model. Finally, we discuss the difference between the backward and the forward approach by comparing optimal strategies and optimal value functions, and see how the gap varies for different values of model parameters.

Keywords
forward dynamic utility; optimal investment; optimal proportional reinsurance; stochastic factor-model; stochastic optimization.

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Coarse Expectations and Non-Fundamental Volatility

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Abstract

To model trading anomalies around major stock market milestones, I assume that traders use coarse Bayesian updating. Asset demands are characterized by discontinuities when expectations shift discretely; these demarcate market regimes. When heterogeneous traders and a linear market maker determine prices, the price discovery process converges in distribution to a Brownian motion. Notably, I obtain this result from stationary expectation errors rather than through exogenous processes. I establish a perfectly non-revealing equilibrium: because traders are different, their aggregated trades generate Brownian prices, yet this price process impedes traders from learning their differences. I then extend the result to equilibrium distributions characterized by heavy tails and highlight policies to curb non-fundamental volatility.

JEL Classification: G14; G41; D84; D91.

Keywords: Non-fundamental volatility; Behavioral finance; Coarse Bayesian updating; Price barriers

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