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A variational approach to innovative technology diffusion

Luca Anzilli
Dipartimento di Scienze dell'Economia,
University of Salento
luca.anzilli@unisalento.it

Extended abstract [1]

In this study, that extends the previous work [1], we propose a variational approach to model the diffusion process of two competing technologies in a given market. We consider space and time diffusion models that assume primarily that the growth in the number of adoptions in each region would vary and the relative number of adoptions would be greater in those regions closest to the regions of innovation origination [4, 5]. Models including spatial effects in technology diffusion have been proposed, e.g., in [2, 3].
We consider the distribution of two (dominant) technologies, previously introduced in the market, observed at a certain instant. From this observation it will appear that some companies in the market will have adopted the first technology, other companies the second one and still others will operate at an intermediate level (even using other technologies). The objective is to identify, on the basis of the observed distribution, what the distribution of the two technologies will be in the long term.

We introduce a suitable variational functional that takes into account the need for market companies to operate at a technology level as close as possible to that of neighbouring companies, the resistance of market operators to vary the technology they possess and the strength with which the two technologies establish themselves on the market. The equilibrium distribution is then determined by minimizing the proposed functional. Some numerical examples for different scenarios are also presented.

This abstract is submitted for the special session on *Optimization, Variational Analysis and Applications* organized by D. Scopelliti and M. Milasi

[1] Speaker: Luca Anzilli, luca.anzilli@unisalento.it.

Keywords
Technology diffusion; Variational model; Minimization.

References

[1] L. Anzilli, A. Farina, Una proposta di applicazione di un funzionale variazionale al problema della diffusione di tecnologie, Working Paper of Dipartimento di Scienze Economiche e Matematico-Statistiche, University of Salento, **58**/**9**(2004), 1–15.

[2] L. De Cesare, A. Di Liddo, A Bolza Optimal Control Problem for Innovation Diffusion, Dynamic Systems and Applications, **9**(2000), 269–280.

[3] M. Lombardi, D. Ritelli, On the Diffusion of an Innovative Technology in the Market, Proceedings of XXI AMASES Meeting, 1997, 373–388.

[4] V. Mahajan, R. A. Peterson, *Models for Innovation Diffusion*, Sage Publication, Beverly Hills, 1985.

[5] K.U. Rao, V.V.N. Kishore, A review of technology diffusion models with special reference to renewable energy technologies, Renewable and sustainable energy reviews, **14**(2010), 1070–1078.

Brezis pseudomonotone bifunctions and quasi equilibrium problems via penalization

Monica Bianchi
Università Cattolica del Sacro Cuore, Milano
monica.bianchi@unicatt.it

Gabor Kassay
Babes-Bolyai University, Cluj
kassay@math.ubbcluj.ro

Pini Rita
Università degli Studi di Milano-Bicocca
rita.pini@unimib.it

Extended abstract [1]

Given a nonempty set C, a bifunction $f : C \times C \to \mathbb{R}$, and a set-valued map $\Phi : C \rightrightarrows C$, the quasi equilibrium problem is defined as follows: find a point $\overline{x} \in \Phi(\overline{x})$ such that

$$f(\overline{x}, y) \geq 0, \quad \forall y \in \Phi(\overline{x}).$$

In this talk we investigate quasi equilibrium problems in a real Banach space under the assumption of Brezis pseudomonotonicity of the function involved. To establish existence results under weak coercivity conditions we replace the quasi equilibrium problem with a sequence of penalized usual equilibrium problems. To deal with the non compact framework, we apply a regularized version of the penalty method. The particular case of set-valued quasi variational inequalities is also considered.

Keywords
Quasi equilibrium problem; Set-valued quasi variational inequality; Brezis pseudomonotonicity; Regularized penalty method; coercivity conditions.

[1] Speaker:[Monica Bianchi, monica.bianchi@unicatt.it].

References

[1] Blum, E., Oettli, W.: From optimization and variational inequalities to equilibrium problems, Math. Student 63, (1994), 123–145

[2] Brezis, H., Nirenberg, L., Stampacchia, G.: A remark on Ky Fan's minimax principle, Boll. Un. Mat. Ital. 6, (1972), 293–300

[3] Fan, Ky: A minimax inequality and applications, Inequalities III Shisha ed. Academic Press, (1972), 103–113

[4] Konnov, I.: Equilibrium formulations of relatively optimization problems, Math. Methods Oper. Res. 90, (2019), 137–152

[5] Mosco, U.: Implicit variational problems and quasi variational inequalities, Lecture Notes in Math. 543, Springer-Verlag, Berlin, (1976), 83–156

Combining approximation and exact penalty in hierarchical programming

Giancarlo Bigi
Dipartimento di Informatica, Università di Pisa
giancarlo.bigi@unipi.it

Lorenzo Lampariello
Dipartimento di Economia Aziendale,
Università degli Studi di Roma Tre,
lorenzo.lampariello@uniroma3.it

Simone Sagratella
Dipartimento di Ingegneria Informatica, Automatica e Gestionale,
Sapienza Università di Roma,
simone.sagratella@diag.uniroma1.it

Extended abstract [1]

Hierarchical programs are optimization problems whose feasible set is implicitly defined as the solution set of another, lower-level, problem. As a major departure from the more general bilevel structures, this talk focuses only on lower-level problems that are non-parametric with respect to the upper level variables. In particular, the minimization of an objective function over the solution set of a lower-level variational inequality is considered (see [3]), which is a special instance of semi-infinite programs [5] and encompasses simple bilevel and equilibrium selection problems as particular cases (see [1, 2, 4]). To tackle this hierarchical problem, a suitable approximated version is introduced. On the one hand, this does not perturb the original (exact) program too much, on the other hand it allows relying on suitable exact penalty approaches whose convergence properties are established [1].

[1] Special session: Optimization, Variational Analysis and Applications by Domenico Scopelliti and Monica Milasi
Corresponding author: Giancarlo Bigi giancarlo.bigi@unipi.it.
Speaker: Giancarlo Bigi giancarlo.bigi@unipi.it.

Keywords
Hierarchical programming; optimization problems with variational inequality constraints; approximation approaches; penalty techniques

References

[1] G. Bigi, L. Lampariello, S.Sagratella, Combining approximation and exact penalty in hierarchical programming, Optimization, (2021) https://doi.org/10.1080/02331934.2021.1939336

[2] S. Dempe, N. Dinh, J. Dutta, T. Pandrit, Simple bilevel programming and extensions, Mathematical Programming **188** (2021) 227–253

[3] J. Outrata, On optimization problems with variational inequality constraints, SIAM Journal on Optimization **4** (1994) 340–357.

[4] S. Sabach, S. Shtern, A first order method for solving convex bilevel optimization problems, SIAM Journal on Optimization **27** (2017) 640–660.

[5] O. Stein, *Bi-level strategies in semi-infinite programming*, Springer, 2013.

Hierarchical Fleet Mix Problems with Risk-Aversion

Riccardo Cambini
Dept. Economics and Management,
University of Pisa
riccardo.cambini@unipi.it

Rossana Riccardi
Dept. Economics and Management,
University of Brescia
rossana.riccardi@unibs.it

Extended abstract [1]

The aim of this paper is to present fleet mix models which consider in an unified framework all of the following crucial aspects: demand uncertainty, demand violation, hierarchical fleet structure, external units for all the hierarchical levels, risk aversion. The proposed models can be a useful strategic management tool aimed to determine an optimal long term strategy.

In particular, the service demand is assumed to be stochastic, depending on a random factor θ. This means that in a first stage, when demand is unknown, decisions on the optimal internal fleet have to be taken on the basis of a suitable estimation of future demand. In a second stage, the possibility of hiring external units for all the considered hierarchical levels can compensate the units shortage in case of demand peaks. Considering an heterogeneous fleet both in the case of internal and external units is a generalized version of previously proposed models. Also quality of service constraints (QOS) are included: for each type of request k a minimum number of fulfilled jobs can be required. In other words, the firm may want to guarantee a certain service level in order to preserve an efficient corporate image. In this light, demand violations are admitted and a penalty cost function is included when the total demand is not completely fulfilled. The objective of these models

[1] Speaker:[Riccardo Cambini - riccardo.cambini@unipi.it].

is to minimize the expected total costs (including penalties) satisfying the quality of service constraints. Moreover, since companies are usually risk averse, a coherent risk measure is introduced to deal with both risk neutral and risk averse operators.

This formulation is a two-stage multi-period stochastic programming model and its deterministic equivalent formulation is deduced with a scenarios approach.

The formulation is comprehensive of a risk neutral and risk averse approach in which the operator can decide the level of risk aversion by moving a parameter $\lambda \in [0, 1]$. In this model a risk neutral agent aims at minimizing the expected value of its cost function by setting $\lambda = 0$ or, in the case of being extremely risk averse, by setting $\lambda = 1$. All the intermediate situations can also be treated with this formulation.

An auxiliary discretized problem is introduced and it is proved that the optimal solution of the original problem can be retrieved by solving the auxiliary problem. This result is able to significantly reduce the computational complexity of the original problem. According to the features of the problem, indeed, the original model belongs to nonlinear integer programming while the auxiliary problem is an integer linear program that can be solved with standard softwares. Notice finally, that the introduction of the auxiliary problem avoids the use of classical approaches either based on branch methods or based on binary variables and the "$big >> 0$" constant.

The results of a computational test are provided in order to validate the model.

Keywords
Hierarchical fleet mix; Stochastic programming; CVaR.

On the extension of continuous quasiconvex functions

Carlo Alberto De Bernardi
Dipartimento di Matematica per le Scienze economiche, finanziarie ed attuariali,
Università Cattolica del Sacro Cuore, Milano,
carloalberto.debernardi@unicatt.it

Extended abstract [1]

Let X be a nontrivial real normed linear space and Y a subspace of X. The natural problem of extending continuous convex real-valued functions from Y to the whole X was studied by several authors in the last two decades, and positive results were obtained under suitable hypotheses involving separability conditions. In [2, 3], the authors consider the more general problem concerning extendability of continuous convex functions from $A \cap Y$ to A, where $A \subset X$ is an open convex set intersecting Y. After a brief review of known results about this problem, we will present recent results, some of which obtained in collaboration with L. Veselý, concerning the corresponding problem for the class of continuous quasiconvex functions.

Keywords
convex function; quasiconvex function; extension

References

[1] C.A. De Bernardi, *A note on the extension of continuous convex functions*, J. Convex Anal., **24** (2017), 333–347.

[2] C.A. De Bernardi and L. Veselý, *Extension of Continuous Convex Functions from Subspaces I*, J. Convex Anal. **21** (2014), 1065–1084.

[1] Speaker:Carlo Alberto De Bernardi, carloalberto.debernardi@unicatt.it.

[3] C.A. De Bernardi and L. Veselý, *Extension of Continuous Convex Functions from Subspaces II*, J. Convex Anal. **22** (2015), 101–116.

[4] C.A. De Bernardi, *On the extension of continuous quasiconvex functions*, J. Optim. Theory Appl., **187** (2020), 421–430.

Session

Optimization, Variational Analysis and Applications. Monica Milasi, Domenico Scopelliti.

A Generalized Ky Fan Minimax Inequality on Finite Dimensional Spaces

Marco Castellani · Massimiliano Giuli
Department of Information Engineering, Computer Science and Mathematics
University of L'Aquila, Italy
marco.castellani@univaq.it · massimiliano.giuli@univaq.it

Extended abstract [1]

Joly and Mosco [1] introduced a class of variational problems involving an extended valued bifunction $\varphi : C \times C \to (-\infty, +\infty]$ which captures the nature of the constraint. This problem asks to

$$\text{find } x \in C \text{ such that } f(x,y) + \varphi(x,y) \geq \varphi(x,x), \text{ for all } y \in C, \qquad (1)$$

where C is a nonempty set, $f : C \times C \to \mathbb{R}$ is a bifunction and the domain of $\varphi(x, \cdot)$

$$D_\varphi(x) = \{y \in C : \varphi(x,y) < +\infty\}$$

is assumed to be nonempty for every $x \in C$.

Aim of this paper is to establish a result concerning existence of solutions for such a class of variational problems defined on a finite-dimensional space. Our approach is based on a Michael selection result for lower semicontinuous set-valued maps [2] in the spirit of [3], combined with a weak coercivity condition which extends an older one proposed by Konnov and Dyabilkin [4] for equilibrium problems and without requiring any assumption of generalized monotonicity. Our result subsumes and extends several similar results which can be obtained by means of suitable choices of the involved functions.

Keywords
Quasiequilibrium problem; Ky Fan minimax inequality; Fixed point; Coercivity condition.

[1] Speaker: [Massimiliano Giuli, massimiliano.giuli@univaq.it]

References

[1] J.-L. Joly, U. Mosco, À propos de l'existence et de la régularité des solutions de certaines inéquations quasi-variationnelles, J. Functional Analysis, **34**(1979), 107–137.

[2] E. Michael, Continuous selections I, Ann. of Math., **63**(1956), 361–382.

[3] M. Castellani, M. Giuli, M. Pappalardo, Existence results for a wide class of equilibrium problems: a general scheme, J. Nonconvex Convex Anal., **21**(2020), 1219–1224.

[4] I.V. Konnov, D.A. Dyabilkin, Nonmonotone equilibrium problems: coercivity conditions and weak regularization, J. Glob. Optim., **49**(2011), 575–577.

Quasiconvex families of functions, with applications to the quasiconvexity of the sum or the infimum of a family of quasiconvex functions

Fabián Flores-Bazán
Department of Mathematical Engineering,
University of Concepción, Concepción, Chile
fflores@ing-mat.udec.cl

Yboon García
Centro de Investigación,
Universidad del Pacífico, Lima, Perú
garcia_yv@up.edu.pe

Nicolas Hadjisavvas
Department of Product and Systems Design Engineering,
University of the Aegean, Syros, Greece
nhad@aegean.gr

Extended abstract [1]

It is well-known that the sum of two quasiconvex functions is not quasiconvex in general, and the same occurs with the minimum. Although apparently these two statements (for the sum or minimum) have nothing in common, they are related, as we show in this talk. To develop our study, the notion of quasiconvex family is introduced, and we establish various characterizations of such a concept: one of them being the quasiconvexity of the pointwise infimum of arbitrary translations of quasiconvex functions in the family; another is the convexity of the union of any two of their sublevel sets; a third one is the quasiconvexity of the sum of the quasiconvex functions, composed with arbitrary nondecreasing functions.

As a by-product, any of the aforementioned characterizations, besides providing quasiconvexity of the sum, also implies the semistrict quasicon-

[1] Speaker: Nicolas Hadjisavvas, nhad@aegean.gr.

vexity of the sum if every function in the family has the same property. Three concrete applications in quasiconvex optimization are presented.

Special section Optimization, variational analysis, and applications. **Organizers:** Monica Milasi, Domenico Scopelliti.

Keywords
quasiconvex function; quasimonotone operator; quasiconvex optimization.

Decomposable Penalty Method for Generalized Game Problems with Joint Constraints

Igor Konnov
Department of System Analysis and Information Technologies,
Kazan Federal University, Russia
konn-igor@ya.ru

Extended abstract [1]

We consider an extension of a noncooperative game problem where players have joint binding constraints. In this case, justification of a generalized equilibrium point needs a reasonable mechanism for attaining this state. We suggest to combine a penalty method together with shares allocation of right-hand sides, which replaces the initial problem with a sequence of the usual Nash equilibrium problems together with an upper level variational inequalities as master problems. We show the convergence of solutions of these auxiliary penalized problems to a solution of the initial game problem under weak coercivity conditions.

This talk is based on the papers [1], [2].

Keywords
Noncooperative games; Generalized equilibrium points; Decomposable penalty method; Variational inequality.

References

[1] I.V. Konnov, Shares allocation methods for generalized game problems with joint constraints, Set-Valued. Variat. Anal., **24** (2016), 499–516.

[2] I.V. Konnov, Decomposable penalty method for generalized game problems with joint constraints, Optimization, https://doi.org/10.1080/02331934.2020.1793153

[1] Speaker:[Igor Konnov, email: konn-igor@ya.ru].

Multistage Stochastic Variational Inequalities and Economic Equilibrium

Monica Milasi
Department of Economics,
University of Messina
mmilasi@unime.it

Domenico Scopelliti
Department of Economics and Management,
University of Pavia,
domenico.scopelliti@unipv.it

Extended abstract [1]

We focus on the study of an economic equilibrium model under uncertainty, introduced by Radner in [2]. The market evolves in a finite sequence of stages and, at each stage, different states of the world are possible. Through this time-uncertain structure, I agents participate in the market; throughout two different structures, forward and spot markets, they consume, trade, and sign contracts. At the beginning agents do not know the possible evolution of the market; the environment is progressively revealed, and, all information is revealed at the final stage. Agents have to make their decisions under uncertainty conditions. In order to capture the essential dynamics of stochastic decision processes, it is needed an approach which encompasses *multistage* models responding to an increasing level of information.
Based on the approach introduced by Rockafellar and Wets in [3], we reformulate the equilibrium problem in terms of a suitable stochastic quasi-variational inequality. Stochastic variational inequalities are a natural extension of the deterministic ones to model and study real-world problems with data affected by some uncertainty and randomness. Here, the considered stochastic variational approach is characterized by the presence of the

[1] Speaker:[Monica Milasi mmilasi@unime.it].

nonanticipativity constraints in the formulation of the problem; these constraints allow us to capture the dynamics of stochastic decision processes in response to an increasing level of information. Stage by stage these constraints impose the measurability with respect to the information field at that stage. Thanks to this characterization, by using variational tools, we give the existence of equilibrium, see e.g. [1]. Finally, we propose an algorithm to compute the equilibrium solution.

Keywords
Equilibrium problem; Nonanticipativity; Multistage; Stochastic variational inequality.

Parallel Session
Optimization, Variational Analysis and Applications
Organizers
Monica Milasi, Domenico Scopelliti

References

[1] M. Milasi, D. Scopelliti, A Stochastic Variational Approach to Study Economic Equilibrium Problems Under Uncertainty, Journal Mathematical Analysis and Applications, 502 **1**(2021).

[2] R. Radner, Existence of Equilibrium of Plans, Prices, and Price Expectations in a Sequence of Markets, Econometrica, 40 **2**(1972), 289–303.

[3] R.T. Rockafellar, R.J-B Wets, Stochastic variational inequalities: single-stage to multistage, Mathematical Programming, Ser.B. 165 **1** (2016), 331–360.

Robustification and scalarization of a parametric vector optimization problem: a noncomponentwise approach

Elisa Caprari
Dipartimento di Scienze Economiche e Aziendali,
Università di Pavia
elisa.caprari@unipv.it

Lorenzo Cerboni Baiardi
Dipartimento di Economia, Statistica e Finanza "Giovanni Anania",
Università della Calabria,
lorenzo.cerboni@unical.it

Elena Molho
Dipartimento di Scienze Economiche e Aziendali,
Università di Pavia,
elena.molho@unipv.it

Extended abstract [1]

The simplest way to introduce robustness in a parametric multiobjective optimization problem is to consider the worst case scenario on each component of the vector-valued objective function. This approach is unduly pessimistic when there is dependence among the uncertainties that affect distinct component of the objective function. Some alternative notions of robust efficiency are based on a reframing of the original problem into a deterministic set-valued optimization setting where the upper quasiorder relation among sets is considered. Under this approach, we study the commutativity of robustification and scalarization by considering the equivalence of the solutions of the scalarization of the set-valued robust counterpart of the uncertain vector optimization problem with the robust counterpart of the scalarization of of the same problem.

[1] Speaker: Elena Molho, elena.molho@unipv.it.

Keywords
Multiobjective optimization; Robust optimization; Set optimization; Scalarization.

References

[1] J. Fliege, R. Werner, Robust multiobjective optimization and applications in portfolio optimization, European Journal of Operational Research, **234**(2014), 422-433, 2014.

[2] C. Gutiérrez, B. Jimenéz, E. Miglierina, E. Molho, Scalarization in set optimization with solid and nonsolid cones, Journal of Global Optimization, **61**(2015), 525-552.

[3] R. Bokrantz, A. Fredriksson, Necessary and sufficient conditions for Pareto efficiency in robust multiobjective optimization, European Journal of Operational Research, **262**(2017), 682-692.

[4] M. Erghott, J. Ide, A. Schobel, Minmax robustness for multi-objective optimization problems, European Journal of Operational Research, **239**(2014), 17-31.

[5] J. Ide, E. Kobis, D. Kuroiwa, A. Schobel, C, Tammer, The relationship between multi-objective robustness concepts and set-valued optimization, Fixed Point Theory and Applications, **83**(2014), 1-20.

A Variational Approach to the Maximization of Preferences Without Numerical Representation

Monica Milasi
Department of Economics,
University of Messina,
mmilasi@unime.it

Domenico Scopelliti
Department of Economics and Management,
University of Pavia,
domenico.scopelliti@unipv.it

Extended abstract [1]

In decision theory, a preference characterizes an individual's attitudes, perceptions, tastes, and inclinations with respect to the alternatives that are the object of choice. Once this is defined, the behavior of the individual considers the preference relation together with any other factor and/or constraint in order to make the best possible decision. This preference is described by means of a binary relation, and the individual does make the best according to it and the constraints in place. In [1], Debreu proved that, under suitable assumptions on the set of alternatives and/or the binary relation, a preference can be represented by means of a real function. Hence, the preference maximization problem can be treated as the maximization problem of functions, and the literature contains several well-known approaches to deal with the resulting optimization problem. In particular, the variational inequalities theory provides powerful and flexible tools to deal with the above described class of problems, in both analysis and computations; see, e.g., [2].
Usually, with preferences there are two different approaches; see, e.g., [3]. The first one is to define a weak preference $\succeq$ and then deduce a strict preference $\succ$. The other approach instead considers a strict preference $\succ$ from which a weak preference relation is deduced. We take the strict preference

[1] Speaker:[Domenico Scopelliti; domenico.scopelliti@unipv.it].

$\succ$ as primitive and we do not require that it is asymmetric and negatively transitive. However, these assumptions are not sufficient to guarantee the completeness and transitivity of the deduced weak preference $\succeq$. In particular, the assumption of completeness means that an individual should be able to compare any two possible alternatives. One can imagine real-life situations in which this assumption does not hold, for instance, when an individual is not able to rank his preferences between two or more choices. For instance, this can occur under uncertain conditions and/or with a lack of information. Furthermore, in many real-world situations, the set of alternatives is vectors, that is, the outcome of a choice involves different features to be evaluated separately. Hence, it is natural to work without the completeness requirement on the individuals' preferences. At the same time, the considered assumptions are not sufficient to guarantee the existence of a utility function representing the preference relation, and then we cannot apply the results known in the literature.
In [4], our aim is to provide a variational approach to study a maximization problem of preferences that cannot be represented by a utility function. In particular, we opportunely characterize the preference maximization problem and we prove some regularity properties on the map of solutions of the relative parametric variational problem. The strength of our approach is that it relies only on the study of the strict upper counter set and the associated normal cone. To the best of our knowledge, there are no studies in which the preferences maximization is studied through a variational approach without numerical representation. We apply the theoretical results to an equilibrium problem under time and uncertainty. This application is constructive for future possible developments and applications.

Keywords
Variational Inequalities; Strict Preference Relation; Equilibrium Problem.

References

[1] G. Debreu, Representation of a preference ordering by a numerical function. In: Thrall M, Davis RC, Coombs C.H (ed) Decision Processes, New York: John Wiley & Sons, (1954), 15–65.

[2] F. Facchinei, J.S. Pang, *Finite-dimensional Variational Inequalities and Complementary Problems*, Springer, New York, 2003.

[3] D.M. Kreps, *A Course in Microeconomics Theory*, Princeton University Press, 1990.

[4] M. Milasi, D. Scopelliti, A Variational Approach to the Maximization of Preferences Without Numerical Representation, Journal of Optimization Theory and Applications, (2021), 1–15.