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BOOK OF ABSTRACTS

Pursuit of One Evader by a Group of Pursuers

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Extended abstract [1]

Many works have been devoted to pursuit-evasion games [1, 2, 3]. In this talk we study a pursuit-evasion differential game with a group of pursuers and a single evader. The control functions of all pursuers and the evader satisfy the integral constraints. The farness between the evader and the closest pursuer when the game is finished is the payoff function of the game. We introduce the value of the game and identify optimal strategies of the pursuers to complete the game. In this game there is no relation between the energy resource of any pursuer and that of the evader.

Keywords
Pursuit-evasion game; Integral constraints.

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Dynamics in energy models: the transportation network and its involving role

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Extended abstract [1]

In this talk we will examine the consequences of including distributed delays in an energy model. In particular, we will present a model that has been developed starting from C.L. Dalgaard and H. Strulik's model [3], a mathematical model of an economy viewed as a transport network for energy. The new model has been developed by C. Bianca et al. [1], modifying the model by C.L. Dalgaard and H. Strulik [3] with the assumption that the energy conservation formula would be influenced by a time delay; they have showed

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that the dynamics of the system is characterized by a delay differential equation. The stability behaviour of the resulting equilibrium for our dynamic system is analyzed including models with Dirac, weak and strong kernels. Applying the Hopf bifurcation theorem we will determine conditions under which limit cycle motion is born in such models. The results indicate that distributed delays have an ambivalent impact on the dynamical behaviour of systems, either stabilizing or destabilizing them. Afterwards, based on V.I. Yukalov et al. [5], C. Bianca et al. [2] have adapted their ideas and proposed a generalization by introducing a logistic-type equation for population with delayed carrying capacity. In their study they have analyzed the consequences of replacing time delays with distributed time delays. C. Bianca et al. [2] have showed that the destructive impact of the agents on the carrying capacity leads the system dynamic behaviour to exhibit stability switches and Hopf bifurcations to occur. Now we will organize a new proposal in this direction.

Keywords
Bifurcation; Energy; Network; Delayed Carrying Capacity.

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Research Dynamics in Western Balkans and the impact of EU enlargements on Science and Innovation by a Network Analysis Model

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Extended abstract

The aim of this work is to examine the publications of international co-authors belonging to some Eastern European Countries between 1994 and 2018. In this context, we want to examine whether the association with the European Union, as countries members or associates, favored Eastern European countries (referred to as East-E) by comparing all publications of these countries with those of EU members. This research question tentatively exposes the advantages in publishing under European Union schemes by the type of affiliation to the European Union itself. To do so, it identifies three subregions a priori: members of the European Union (East-EU); being an affiliated country to EU research schemes (East-AC); or neither (East-Ext). This is tested at different levels: number of publications (articles co-authored with at least one East-E presence); centrality of a given country in the global network of collaborations. The findings show that to be EU member or associated countries does play a positive role, although national differences within these different types of affiliation are more relevant than those between the three sub-regions. Findings suggest further research directed at understanding national policies concerning research, and how the European Union might consider its contribution in the wider European Research Area. These findings also suggest further research concerning the future of Eastern Europe, especially in a possible scenario of "two-speeds integration" of the European Union and the European Research Area.

Keywords
Social Network Analysis, International Collaborations, Western Balkans and EU, Substitution Effect.

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DYNAMIC SEARCH ON THE EDGES OF THE GRAPH

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In this paper the dynamic game of catching a point moving on the geometric (conjugate) graph (with rectifiable edges) in space R^d is considered. Suppose that the graph is connected and n and l are the numbers of its vertices and edges, respectively, the length of all edges equals to 1. The game involves two players moving on the edges of the graph: **P** - the team of pursuers controlling the movement of points $P_1, P_2, ..., P_m$ and Q - the evader controlling the point [1], [5], [6]. The maximum speed off all players is equal to 1.

Further, we determine the natural number $N(G)$ [2], [3], [4]. The smallest of the numbers m that **P** wins in the game is denoted by $N(G)$. If G is the free, then $N(G) = 1$, if the graph has at least one cycle, then $N(G) \geq 2$. It is obvious that $N(G)$ is exist and $N(G) \leq n - 1$.

Now, we take the point $A \in R^d$ that does not belong to the graph and connect this point with some vertex of the graph G. By connecting the point A with the vertices of the graph G, we will obtain the a geometric graph G_1 with $n+1$ vertices and $l+1$ edges, and having the properly $V(G) \subset V(G_1)$, $E(G) \subset E(G_1)$. Assume that the length of the edge coming out of the point A is equal to 1.

We determine the natural number $k_1 = N(G_1)$ by looking at a game involving **P** - a team of pursuers and the evader Q along the edges of the graph G_1.

Similarly, by connecting the point A with vertices of the graph G_1, we will create a geometric graph G_2 with the number of vertices $n + 1$, the number of edges $l + 2$. And, it holds: $V(G) \subset V(G_1) \subset V(G_2)$, $E(G) \subset$

$E(G_1) \subset E(G_2)$.

We determine the natural number $k_2 = N(G_2)$ by looking pursuit-evasion game on the edges of the graph G_2.

Continuing this process, we will create a geometric graph G_i with the number of vertices $n+1$, the number of edges $l+i$. And, it holds: $V(G) \subset V(G_1) \subset ... \subset V(G_i)$, $E(G) \subset E(G_1) \subset ... \subset E(G_i)$, $i = 1, 2, ..., n$.

We determine the natural number $k_i = N(G_i), i = 1, 2, ..., n$ by looking pursuit-evasion game on the edges of these graphs.

Consequently, we obtain the sequence $k_1, k_2, ..., k_n$. Denote by $K = \{k_1, k_2, ..., k_n\}$ this sequence.

Let us give some properties of K.

1. If $N(G) = 1$, then $K = \{1, 2, 2, ..., 2\}$.
2. If $N(G) > 1$, then $min\{k_1, k_2, ..., k_n\} = 2$.
3. $k_1 = N(G)$.
4. $k_n = 2$.
5. $max\{k_1, k_2, ..., k_n\} \leq k_1 + 1$.
6. $k_{i+1} - k_i \in \{-1, 0, 1\}, k_i \in K, i = 1, 2, ..., n-1$.
7. $|\{i | k_{i+1} - k_i = -1, k_i \in K, i = 1, 2, ..., n-1\}| \leq 1$, where $|\Omega|$ - the number of elements of the set Ω.
8. $|\{i | k_{i+1} - k_i = 1, k_i \in K, i = 1, 2, ..., n-1\}| = max\{k_1, k_2, ..., k_n\} - min\{k_1, k_2, ..., k_n\}$,.

Keywords
A geometric graph; Dynamic game; The team of pursuers; The evader; Pursuit-evasion game.

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PURSUIT-EVASION GAME ON THE GRAPH OF 1-SKELETON OF THE ICOSAHEDRON

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Let G denote the graph of 1-skeleton of the regular polyhedrons with icosahedron in Euclidian space R^3 [1], [3]. The team of Pursuers $\mathbf{P} = \{P_1, P_2, ..., P_m\}$ and one Evader Q moving along play a pursuit-evasion differential game. Speed of Q doesn't exceed 1 while maximal speed of point P_i equals $\rho_i, \rho_i \leq 1, i = 1, 2, ..., m$, $1 \geq \rho_1 \geq \rho_2 \geq ... \geq \rho_m > 0$. The process of pursuit-evasion begins from initial positions $\mathbf{P}(0) = \{P_1(0), P_2(0), ..., P_m(0)\}$, $Q(0)$. If one of the players chooses concrete strategy and other chooses arbitrary control function the $\mathbf{P}(t) = \{P_1(t), P_2(t), ..., P_m(t)\}, Q(t), t \geq 0$ then corresponding trajectories will be generated. The aim of the team of pursuers is to reach the equality $P_i(T) = Q(T)$ for some $i = 1, 2, ..., m$, $T \geq 0$ for any initial positions. The aim of evader is opposite, i.e. to hold the condition $P_i(t) = Q(t)$ for all $i = 1, 2, ..., m$ and $t, t \geq 0$ for some initial position (see [2]-[4]).

Obviously, if m is great enough then the team of Pursuers can win the game. The least value of m that m Pursuers win the game, will be denoted by $N(G)$.

Theorem 1. If $\rho_1 = 1, \rho_2 > 0, \rho_3 > 0$ then on the Pursuit-Evasion game, the team of Pursuers wins.

Theorem 2. If $\rho_1 = 1, \rho_2 = 10$ then on the Pursuit-Evasion game, the Evader wins.

Theorem 3. If $\rho_1 \geq \frac{2}{3}, \rho_2 \geq \frac{2}{3}, \rho_3 \geq \frac{2}{3}$ then on the Pursuit-Evasion game, the team of Pursuers wins.

Theorem 4. If $\rho_1 < 1, \rho_2 < \frac{1}{2}, \rho_3 < \frac{1}{2}$ then on the Pursuit-Evasion game, the Evader wins.

Keywords
The regular polyhedron; Icosahedron; The team of pursuers; The evader; Pursuit-evasion game.

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Pursuit differential game problem on dodecahedron

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Extended abstract [1]

The present paper is devoted to a pursuit differential game problem on 1-skeleton of dodecahedron. Let the group of three pursuers $P = \{P_1, P_2, P_3\}$ and one evader Q move along the edges (1-skeleton) of dodecahedron $A_1...A_5$-$C_1...C_{10}B_1...B_5$. Without any loss of generality we assume that the lengths of edges of dodecahedron are equal to 1. It is assumed that each player knows positions of other players at the present time t. Moreover, pursuers know the evader's velocity at the present time t as well. We use $P_i(t)$, $i = 1, 2, 3$, and $Q(t)$ to denote the positions of pursuers and evader at the time t.

It is assumed that $P_i(0) \neq Q(0)$, $i = 1, 2, 3$. We'll construct strategies for the pursuers to complete the game for any behavior of the evader. We denote the maximum speed of evader and i-th pursuer by σ and ρ_i, respectively. Without restriction of generality we assume that $\sigma = 1$.

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Definition. *If there exist strategies of pursuers such that for any control of evader $P_i(\tau) = Q(\tau)$ for some $i \in \{1, 2, 3\}$ and $\tau > 0$, then we say that the group of pursuers wins the game.*

The following statement is the main result of the paper.

Theorem. *If $\frac{2}{3} \leq \rho_1 \leq 1$, $\frac{2}{3} \leq \rho_2 \leq 1$, and $\rho_3 > 0$, then the group of pursuers P wins the game. Moreover, the game is completed by the time $\frac{25}{2\rho_3}$.*

There are many articles, devoted to pursuit and evasion differential game problems on the edge graph of polyhedron (see, for example, $[1-4]$). In the present paper, a pursuit differential game problem on the 1-skeleton of dodecahedron is studied for the first time when the speeds of pursuers are less than that of the evader.

Keywords
Pursuit differential game; evader; pursuer; dodecahedron; strategy.

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ALGORITHM OF SOLUTION OF ONE GAME OF TWO PERSONS

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Extended abstract [1]

Let $T = [0, t^*]$. The behavior of a system controlled by two players (participants), is described by the differential equation [1,2]:

$$\dot{x} = Ax + bu + dv, \quad x(0) = x_0, \tag{1}$$

where $x = x(t) = (x_1(t), x_2(t), ..., x_n(t))'$ is the state vector of system at the current time t; u and v are the control parameters of the first and second players, respectively, at the time t; A is an $n \times n$ matrix, b, d, x_0 are given n-vectors.

Piecewise constant function $u(\cdot) = (u(t), t \in T)$ being continuous from the right and satisfying the inequalities $f_* \leq u(t) \leq f^*$, $t \in T$, is called the control of the first player, where f_*, f^* are given numbers.

Impulse function [2] $v(\cdot) = (v(t), t \in T)$ with set of quantization

$$\tau = \{t_1, t_2, ..., t_l\}, \quad 0 = t_1 < t_2 < ... < t_l < t_{l+1} = t^* \quad (l \geq m),$$

satisfying the inequalities $g_*(t) \leq v(t) \leq g^*(t)$, $t \in T$, is called the control of the second player, where $g_*(t), g^*(t)$, $t \in T$, are given impulse functions with the set quantization τ, and t^* is a given positive number.

According to the theory of differential equations, for each pair $\{u(\cdot), v(\cdot)\}$ of the players' controls, there corresponds the only continuous solution $x(\cdot) = (x(t), t \in T)$ of equation (1), the trajectory of the dynamic systems.

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Let H, g, and c be given $m \times n$ constant matrix, m vector, and n vector, respectively, U, V be the sets of control functions of the first and second players, respectively. The terminal set and the objective functional are defined respectively by equations

$$M = \{x \in \mathbb{R}^n \mid Hx = g\}, \quad J(u(\cdot), v(\cdot)) = c'x(t^*).$$

Consider the following game problem. The first player chooses a control $u(\cdot) = (u(t), t \in T)$, $f_* \leq u(t) \leq f^*$, $t \in T$, and then knowing this control the second player chooses a control $v(\cdot) = (v(t), t \in T)$, $g_*(t) \leq v(t) \leq g^*(t)$, $t \in T$.

The goal of the first player is not to bring the trajectory of system (1) into the set M at the time t^* by selecting a control $u^0(\cdot) = (u^0(t), t \in T)$, if this possible, else to maximize the functional $\min_{v(\cdot)\in V} J(u(\cdot), v(\cdot))$. The goal of the second player is to bring the trajectory of system (1) from the point x_0 to the set M at t^* by choosing a control $v^0(\cdot) = (v^0(t), t \in T)$ and minimize the functional $J(u(\cdot), v(\cdot))$.

In the present paper, following [3] this problem has been investigated by using the method of special nonsmooth problem optimization.

Using the connection between these problems an algorithm has been developed for solving the stated problem. The algorithm is based on comparing the values of special controls of players in the dual problem to the special nonsmooth optimization problem.

Keywords
differential equations; a game; players; nonsmooth problem.

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Differential games of many players on the 1-skeleton of a regular simplex

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Extended abstract [1]

Pursuit and evasion differential games are studied on 1-skeleton Σ^d of the regular simplex of dimension d, which is defined as a subset of the Euclidean space $\mathbb{R}^d$ by the following relations

$$\sum z_j = \frac{1}{\sqrt{2}}, \quad z_j \geq 0, \; j = 1, 2, ..., d+1.$$

Its edges of length 1 form a complete graph Σ^d with $d+1$ vertices. Let the group of n pursuers and one evader move on Σ^d according to the following equations

$$\dot{x}_i = u_i, \qquad x_i(0) = x_{i0}, \quad i = 1, 2, ..., n, \tag{1}$$

$$\dot{y} = v, \qquad y(0) = y_0, \tag{2}$$

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where $x_i, y, x_{i0}, y_0 \in \Sigma^d$, $u_i, v \in \mathbb{R}^d$, $x_i(t)$ and $y(t)$ are the states, and u_i, v are control parameters of ith pursuer and evader, respectively. It is assumed that $|v| \leq 1$, $|u_i| \leq \rho_i$, $i = 1, 2, ..., n$, where $\rho_i < 1$, $i = 1, 2, ..., n$, are given positive numbers. It is assumed that all the players move only along the 1-skeleton of the regular simplex Σ^d.

Pursuers use information about the values $x_1(t), ..., x_n(t), y(t)$ and $v(t)$ at current time t to construct their strategies. The evader uses information about $x_1(t), ..., x_n(t), y(t)$ to construct his strategy.

Definition 1 *If there exist strategies of pursuers such that for any control of evader $x_i(\tau) = y(\tau)$ for some $i \in \{1, 2, ..., n\}$ and $\tau > 0$, then we say that pursuit can be completed in the game.*

Let $\frac{1}{2}\sigma \leq \rho_i < \sigma$, $i = 1, 2, ..., k$, and $\rho_i < \frac{1}{2}\sigma$, $i = (k+1), ..., n$ for some integer $k \geq 0$.

Theorem 2 *If either (i) $n = k$ and $n + k > d$ or (ii) $n > k$ and $n + k \geq d$, then pursuit can be completed in the game.*

Definition 3 *If there exists a strategy of evader such that for any control of pursuers $x_i(t) \neq y(t)$ for all $i = 1, 2, ..., n$, and $t > 0$, then we say that evasion is possible in the game.*

Theorem 4 *If either (i) $n = k$ and $n + k \leq d$ or (ii) $n > k$ and $n + k < d$, then evasion is possible.*

Keywords
Pursuit differential game; evasion differential game; 1-skeleton of simplex; strategy.

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The optimal control problem of an ensemble of trajectories of differential inclusion with delay under terminal constraints

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Extended abstract [1]

In the modern theory of dynamic control systems, methods of multivalued analysis and the mathematical apparatus of the theory of differential inclusions are widely used [1]. Differential inclusions with a control parameter can be used to study non-deterministic models of control systems. Methods for controlling ensembles (bundles) of trajectories of differential inclusions have effective applications in constructing problem an optimal control that gives a guaranteed value of the quality criterion [2, 3]. The problems of controlling bundles of trajectories also arise in dynamic models of conflict situations – in differential games [4].

Consider a controllable differential inclusion with delays of the form [3]

$$\frac{dx}{dt} \in A(t)x + \sum_{i=1}^{k} A_i(t)x(t-h_i) + b(t,u), \tag{1}$$

where $t \in [t_0, t_1],\ x(t) = \varphi_0(t),\ t \in [t_0 - \max\limits_{i=\overline{1,k}} h_i, t_0],\ u \in V.$

Let: the elements of $n \times n$ -matrices $A(t)$ and $A_i(t)$, $i = \overline{1,k}$ are summable on $T = [t_0, t_1]$; multivalued mapping $(t,u) \to b(t,u) \in R^n$ with convex

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compact values, measurable by $t \in T$; support function $C(b(t,u),\psi)$ convex by $u \in V$; $\|b(t,u)\| \leq \beta(t)$, $\beta(\cdot) \in L_1(T)$; V - is a convex compact R^m; $\varphi_0(\cdot) \in C^n(T_0)$. Then: the set of admissible controls $U(T)$, consisting of all measurable functions $u = u(t) \in V, t \in T$, is weakly compact in $L_2^m(T)$; the set of absolutely continuous on T trajectories of system (1) $H(u,\varphi_0)$ is a non-empty compact set in $C^n(T_1)$; the ensemble of trajectories $t \to X(t,\, u, \varphi_0) = \{\xi \in R^n : \xi = x(t),\ x(\cdot) \in H(u,\varphi_0)\}$, $t \in T$ is a continuous multivalued mapping whose values are convex compact sets in R^n, and the support function $C(X(t,u,\varphi_0),\psi)$ is convex by $u(\cdot) \in U(T)$.

Consider the problem of terminal control of an ensemble of trajectories of system (1):

$$C(X(t_1,u,\varphi_0),l_0) \to \min,\ C(X(t_1,u,\varphi_0),l_i) \leq q_i,\ i = \overline{1,k},\ u(\cdot) \in U(T). \tag{2}$$

Definition. Problem (2) is called regular if there exists $\tilde{u}(\cdot) \in U(T)$ such that $C(X(t_1,\tilde{u},\varphi_0),l_i) < q_i$, $i = \overline{1,k}$.

Theorem. Let problem (2) is regular. Then for the optimality of $u^*(\cdot) \in U(T)$ it is necessary and sufficient the existence of $\lambda^* = (\lambda_1^*, ..., \lambda_k^*)$, $\lambda_i^* \geq 0$, $i = \overline{1,k}$ such that the pair $\{u^*(\cdot), \lambda^*\}$ constitutes the saddle point of the Lagrange functional

$$L(u(\cdot),\lambda) = C(X(t_1,u,\varphi_0),l_0) + \sum_{i=1}^{k} \lambda_i (C(X(t_1,u,\varphi_0),l_i) - q_i).$$

Necessary and sufficient optimality conditions of the following form were obtained from this theorem for the regular problem (2)

$$\min_{v \in V}[C(b(t,v),\psi^0(t)) + \sum_{i=1}^{k} C(b(t,v),\psi_i^*(t))] =$$

$$= C(b(t,u^*(t)),\psi^0(t)) + \sum_{i=1}^{k} C(b(t,u^*(t)),\psi_i^*(t)), t \in T, \tag{3}$$

where $\psi^0(t) = \psi(t,l_0)$, $\psi_i^*(t) = \psi(t,\lambda_i^* l_i)$, function $\psi(t,l)$ is the solution of the system:

$$\dot{\psi} = -A'(t)\psi(t) - \sum_{i=1}^{k} A_i'(t+h_i)\psi(t+h_i),\ \psi(t_1) = l. \tag{4}$$

It follows from the obtained result that in order to construct optimal control in the considered problem, one should first find a solution to the system (4) and then solve a parameterized finite-dimensional optimization problem (3).

Keywords
differential inclusion; ensemble of trajectories; optimal control; terminal control.

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Pursuit and Evasion Differential Games for an Infinite System of 2-systems of Differential Equations

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Extended abstract [1]

We study a differential game for the following system

$$\begin{aligned} \dot{x}_i &= -\alpha_i x_i - \beta_i y_i + u_{i1} - v_{i1}, \qquad x_i(0) = x_{i0}, \\ \dot{y}_i &= \ \ \beta_i x_i - \alpha_i y_i + u_{i2} - v_{i2}, \qquad y_i(0) = y_{i0}, \end{aligned} \tag{1}$$

in Hilbert space l_2, where α_i, β_i are real numbers, $\alpha_i \geq 0$, $(x_{10}, x_{20}, \ldots)$, $(y_{10}, y_{20}, \ldots) \in l_2$, $u = (u_1, u_2, ...)$ with $u_i = (u_{i1}, u_{i2})$ and $v = (v_1, v_2, ...)$ with $v_i = (v_{i1}, v_{i2})$, $i = 1, 2, ...$, are pursuer's and evader's control parameters. We assume that $0 \leq t \leq T$, where T is a sufficiently large number, and $z_0 = (x_{10}, y_{10}, x_{20}, y_{20}, \ldots) \neq 0$.

Let ρ and σ be given positive numbers. An admissible control of pursuer (evader) is a function $u(t) = (u_1(t), u_2(t), \ldots)$ $(v(t) = (v_1(t), v_2(t), \ldots))$, $t \in [0, T]$, whose coordinates $u_i(t)$ $(v_i(t))$ are measurable and satisfy the condition

$$\sum_{i=1}^{\infty}(u_{i1}^2(t) + u_{i2}^2(t)) \leq \rho^2, \quad \left(\sum_{i=1}^{\infty}(v_{i1}^2(t) + v_{i2}^2(t)) \leq \sigma^2\right), \ 0 \leq t \leq T.$$

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Let $\rho > \sigma$, $z_i(t) = (x_i(t), y_i(t))$, $z_{i0} = (x_{i0}, y_{i0})$. We call a number θ a guaranteed pursuit time if for some strategy of pursuer and for any admissible control of the evader, $z(t') = 0$ at some t', $0 \le t' \le \theta$, where $z(t) = (z_1(t), z_2(t), \ldots)$ is the solution of the initial value problem (1). The pursuer is interested in minimizing the guaranteed pursuit time.

A number τ is called a guaranteed evasion time if for any number τ', $0 \le \tau' < \tau$, we can construct an admissible control for the evader such that for any admissible control of the pursuer, we have $z(t) \neq 0$ for all $0 \le t \le \tau'$ and $i = 1, 2, \ldots$. The evader is interested in minimizing the guaranteed evasion time. Problem is to find an equation for a guaranteed pursuit time and a guaranteed evasion time in the game (1).

Theorem 1. For the initial state $z_0 = (z_{10}, z_{20}, \cdots)$, the number θ that satisfy the equation

$$\sum_{\alpha_i > 0} \frac{\alpha_i^2 |z_{i0}|^2}{\sinh^2(\alpha_i \theta)} + \frac{1}{\theta^2} \sum_{\alpha_i = 0} |z_{i0}|^2 = (\rho - \sigma)^2$$

is a guaranteed pursuit time in the game (1).

Theorem 2. For the initial state $z_0 = (z_{10}, z_{20}, \cdots)$, the number

$$\tau = \sup_i \tau_i, \quad \tau_i = \begin{cases} \frac{1}{\alpha_i} \ln \left(\frac{\alpha_i |z_{i0}|}{\rho - \sigma} + 1 \right), & \alpha_i > 0 \\ \frac{|z_{i0}|}{\rho - \sigma}, & \alpha_i = 0 \end{cases}$$

is a guaranteed evasion time in game (1).

Keywords
Differential game, pursuit, control, strategy, infinite system of differential equations, geometric constraint.

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Optimal Lockdown Strategies driven by Socioeconomic Costs

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Extended abstract [1]

The recent pandemic of COVID-19 that spread all over the world has put Governments at hard testing, because they have to manage a global health crisis with dramatic effects on both human lives and economies. In this research, we modify a classical SIR model to better adapt to the dynamics

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of the COVID-19. We propose the heterogeneous SQAIRD model where COVID-19 spreads over a population of economic agents, namely: the elderly, adults and young people. Within each demographic group, the population is divided in five subgroups: Susceptible (S), Quarantined (Q), Asymptomatic (A), Infected (I), Recovered (R), and Dead (D). The group of Susceptibles are those who are exposed to the virus. The group indicated by Q is the group Quarantined (i.e. isolated at home), by law or by their will. The group Asymptomatic is a group of people who caught the virus, but did not show evident symptoms. These people can behave like a susceptible but they can infect other people as well. Then they can either recover (then shift into the group of Recovered) or develop symptoms, then shifting in the group of Infected with symptoms. An Infected, instead, can either recover and shift into Recovered group or die (then shift into the Dead group). Once a person shift into the Recovered or Dead groups, it cannot be infected anymore (we assume that a person that recovers, develops immunity and cannot be reinfected). We design and simulate an optimal control problem faced by a Government, where its objective is to minimize the costs generated by the pandemics using as control a compulsory quarantine measure (that is, a lockdown). We first analyze the problem from a theoretical perspective, claiming that different lockdown policies (total lockdown, no lockdown or partial lockdown) may justified by different cost structures of the economies. We analyze a particular cost structure (convex costs) and simulate a targeted optimal policy vs. a uniform optimal policy, by dividing the whole population in three demographic groups (young, adults and old). We also simulate the dynamic of the pandemic with no policy implemented.

Keywords
Epidemic process; SIR model; Quarantine; Optimal control.

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Effectiveness of protection measures in clustered epidemic model with intralevel migration

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Extended abstract [1]

Nowadays, epidemic modelling provides an appropriate tool for describing the propagation of biological viruses in human or animal populations, information in social networks and malicious software in computer or ad hoc networks. The current study represents a hierarchical epidemic model that describes the propagation of a pathogen in the clustered human population. Estimation of Novel coronavirus spreading worldwide leads to the idea

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of the hierarchical structure of the epidemic process. Thus, the propagation process is divided into several clusters. On each cluster, the pathogen propagation process is based on the Susceptible-Exposed-Infected-Recovered (SEIR) model. We formulate the modified model of transmission of the infected individuals between the clusters. The control of pathogen spreading can be seen as an optimal control problem where a tradeoff exists between the cost of active virus propagation and the design of the appropriate quarantine or pharmaceutical measures. Its network defines each cluster in the hierarchical system.

We estimate the effectiveness of protection measures within clusters and between clusters of the population. Intralevel control is defined by increasing the proportion of the population in a Quarantine and increasing the effectiveness of the treatment of infected agents. In contrast, inter-level control impacts the intensity of migration rate between clusters. Thus, we compare pharmaceutical interventions with several types of non-pharmaceutical ones. By series of numerical experiments, we demonstrate the network structure's influence on the interaction between clusters and inside clusters in a hierarchical epidemic model. The series of numerical experiments are corroborated the obtained results.

Keywords
Epidemic process; SIR model; Quarantine; Optimal control.

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Differential Games with Grönwall Type Constraints on Controls

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Extended abstract [1]

In the present work, a simple motion pursuit-evasion differential game of one pursuer and one evader is studied. The strategies of players are constructed and solvability conditions of the pursuit-evasion game are given.

We propose a new set of controls of pursuer and evader described by generalized Grönwall type constraints

$$|u(t)| \leq \rho_0 + \rho_1 t + k \int\limits_0^t |u(s)| ds, \text{ a.e. } t \geq 0,$$

$$|v(t)| \leq \sigma_0 + \sigma_1 t + k \int\limits_0^t |v(s)| ds, \text{ a.e. } t \geq 0,$$

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respectively, where $\rho_0>0$, $\sigma_0 > 0$, $\rho_1 > 0$, $\sigma_1 > 0$, $k \geq 0$.

Let dynamics of pursuer P and evader E be described by the equations

$$P\text{:}\ \dot{x} = u, \quad x(0) = x_0,$$

$$E\text{:}\ \dot{y} = v, \quad y(0) = y_0,$$

where $x, y, x_0, y_0, u, v \in \mathbb{R}^n, n \geq 1, x_0 \neq y_0$.

The aim of the pursuer is capture, i.e., to reach $x(t) = y(t)$ and the evader struggles to avoid an encounter, i.e., to achieve $x(t) \neq y(t)$ for all $t \geq 0$, and in the opposite case, to postpone the instant of encounter as long as possible.

Definition 1. If $\delta_0 \geq 0$, $\delta_1 \geq 0$, then the function

$$\mathbf{u}_{Gr}(t, v) = v - \lambda_{Gr}(t, v)\xi_0 \tag{1}$$

is called a Π_{Gr}-strategy of the pursuer in the pursuit game, where $\lambda_{Gr}(t, v) = \langle v, \xi_0\rangle + \sqrt{\langle v, \xi_0\rangle^2 + \varphi^2(t) - |v|^2}$, $\xi_0 = z_0/|z_0|$, $\delta_0 = \rho_0 - \sigma_0$, $\delta_1 = \rho_1 - \sigma_1$, $\varphi(t) = \frac{\rho_1}{k}(e^{kt} - 1) + \rho_0 e^{kt}$, $\varphi(0) = \rho_0$.

Proposition 1. Assume that $\delta_0 \geq 0$, $\delta_1 > 0$ or $\delta_0 > 0$, $\delta_1 \geq 0$. Then there exists at least one positive root of the equation

$$e^{kt} = At + B \tag{2}$$

with respect to t, where $A = \frac{k\delta_1}{\delta_1 + k\delta_0}$, $B = 1 + \frac{k^2|z_0|}{\delta_1 + k\delta_0}$. The smallest root of equation (2) is called call a guaranteed pursuit time and denoted by T_{Gr}.

Theorem 1. Let Proposition 1 is valid. Then the strategy (1) guarantees completion of pursuit on the time interval $[0, T_{Gr}]$.

To solve the evasion problem we will propose a strategy of the evader.

Definition 2. We call a strategy of the evader control function

$$\mathbf{v}_{Gr}(t) = -\psi(t)\xi_0, \ t \geq 0. \tag{3}$$

where $\psi(t) = \frac{\sigma_1}{k}(e^{kt} - 1) + \sigma_0 e^{kt}$, $\psi(0) = \sigma_0$.

Theorem 2. If $\delta_0 \leq 0$, $\delta_1 \leq 0$, then the strategy (3) is winning for the evader in the evasion game.

Keywords
Differential game; pursuit; evasion; Grönwall type constraint; strategy.

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Differential game: life line in the Pontryagin control example

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Extended abstract [1]

In this work, it is studied the problem of R.Isaacs called "Life line" game in $\mathbb{R}^n$. Here pursuit problem will be solved by parallel pursuit strategy.

Let dynamics of the players be described by the equations

$$P\text{:}\ \ \ddot{x} - a\dot{x} = u,\ x(0) = x_0,\ \dot{x}(0) = x_1,\ \ |u(t)| \leq \alpha \ \ \text{a.e. } t \geq 0,$$

$$E\text{:}\ \ \ddot{y} - a\dot{y} = v,\ y(0) = y_0,\ \dot{y}(0) = y_1,\ \ |v(t)| \leq \beta \ \ \text{a.e. } t \geq 0,$$

where $x, y, u, v \in \mathbb{R}^n,\ n \geq 2,\ a \neq 0,\ x_0 \neq y_0,\ x_1 = y_1,\ \alpha > 0,\ \beta \geq 0.$

P aims to catch E i.e. to realize $x(t) = y(t)$ for some $t > 0$, while E stays in the zone $\mathbb{R}^n \setminus M$. The aim of E is to reach the zone M before being caught by P or to keep $x(t) \neq y(t)$ for all $t, t \geq 0$. Notice that M doesn't restrict motion of P and $y_0 \notin M$.

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Let $z = x - y,\ z_0 = x_0 - y_0$.

Definition 1. If $\alpha \geq \beta$ for all $t \geq 0$, then the function

$$\mathbf{u}(z_0, v) = v - \lambda(z_0, v)\xi_0,$$

is called Π_G-strategy for P, where $\lambda(z_0, v) = \langle v, \xi_0 \rangle + \sqrt{\langle v, \xi_0 \rangle^2 + \alpha^2 - |v|^2}$, $\xi_0 = z_0/|z_0|$.

Definition 2. The smallest positive root of the equation $e^{at} = at + 1 + \frac{a^2|z_0|}{\alpha - \beta}$ is called a guaranteed pursuit time and denoted by T.

Theorem 1. a) Let $\alpha > \beta$. Then Π_G-strategy is winning for P on time interval $[0, T]$; b) If $\alpha \leq \beta$, then for any control of P, the strategy $v(t) = -\beta\xi_0$ is winning for E i.e. $|z(t)| \geq |z_0|$ on time interval $[0, \infty)$.

Let $\alpha > \beta,\ \varphi(t) = \frac{1}{a}(e^{at} - 1)$. Then for the pair $(x(t), y(t)))$ we construct the set

$$W(t) = W(x(t), y(t)) = \{w : \beta|w - x(t)| \geq \alpha|w - y(t)|\},$$

$$W(0) = W(x_0, y_0) = \{w : \beta|w - x_0| \geq \alpha|w - y_0|\}.$$

It is clear that $y(t) \in W(t)$ for all $t \in [0, T]$.

Theorem 2. The multi-valued mapping $W(t) - \varphi(t)x_1$ is monotone decreasing in t, $t \in [0, T]$, i.e. $W(t_1) - \varphi(t_1)x_1 \supset W(t_2) - \varphi(t_2)x_1$ when $0 \leq t_1 \leq t_2$ for any $t_1, t_2 \in [0, T]$.

Theorem 3. If $W_P \bigcap M = \emptyset$, then the Π_G-strategy is winning for P, where $W_P = \{W(0) + \varphi(t)x_1 : t \in [0, T]\}$.

Theorem 4. If $W_E \bigcap M \neq \emptyset$, then there exists some control of E which is winning, where

$$W_E = \left\{\tilde{\omega} : \tilde{\omega} = \varphi(\tau)x_1 + \frac{\beta(\omega - y_0)}{a|\omega - y_0|}(\varphi(\tau) - \tau) + y_0,\ y(\tau) = \tilde{\omega},\ \omega \in W(0)\right\}.$$

Keywords

Differential game; pursuit; evasion; strategy; life line.

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Pursuit of One Evader by a Group of Pursuers

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Extended abstract [1]

Many works have been devoted to pursuit-evasion games [1, 2, 3]. In this talk we study a pursuit-evasion differential game with a group of pursuers and a single evader. The control functions of all pursuers and the evader satisfy the integral constraints. The farness between the evader and the closest pursuer when the game is finished is the payoff function of the game. We introduce the value of the game and identify optimal strategies of the pursuers to complete the game. In this game there is no relation between the energy resource of any pursuer and that of the evader.

Keywords
Pursuit-evasion game; Integral constraints.

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Representation of the solution of one dynamical system on the plane

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Abstract. In this paper, we consider a system of second-order nonlinear differential equations with a special right-hand side, exactly, the linear part and a third-order polynomial of a special form. It is shown that for some relations between the parameters there is a second-order curve which trajectories leaving the points of this curve remain in the same place. Thus, the curve is invariant respect to the given system. Moreover, this system is invariant under a non-degenerate linear transformation of variables. We prove that the form of this curve depends on the relations between the parameters and the eigenvalues of the matrix.

Keywords. Dynamic system, ellipse, hyperbola, Hess system, polar coordinate system.

Let the dynamic system have the form

$$\dot{x} = Ax + (a \cdot x)x + (x^T Bx)x, \quad x = (x_1, \dots, x_n)^T \in R^n, \tag{1}$$

$$x(0) = x^0, \tag{2}$$

where the A, B-constant matrices order $n \times n$.

The family of system (1) is invariant relatively linear transformations of the form

$$x = Ly$$

where the L-non-degenerate matrix of dimension $n \times n$

Actually,

$$L\dot{y} = ALy + (a \cdot Ly)Ly + (Ly)^T BLyLy,$$

$$\dot{y} = L^{-1}ALy + (L^T a \cdot y)L^{-1}Ly + L^{-1}(Ly)^T BLy \cdot Ly,$$

$$\dot{y} = L^{-1}ALy + (L^T a \cdot y)y + (Ly)^T BLy \cdot L^{-1}Ly,$$

And finally we get

$$\dot{y} = L^{-1}ALy + (L^T a \cdot y)y + y^T L^T BLy \cdot y$$

It should be noted that, the class of system (1) has a more property of invariance, if the $\bar{x} -$ singular point, we again obtain a system of the form (1), when transferring the origin $\bar{x} -$ to the point $x = = y + \bar{x}$. Specifically,

$$\dot{y} = \tilde{A}y + (\tilde{a}y)y + (y^T By)y, \qquad y = (y_1, y_2)^T \in R^2,$$

where the $\tilde{A} = A + \bar{x} \otimes a + (a, \bar{x})E\tilde{A}y + (\tilde{a}y) + \bar{x}^T B\bar{x}E + \bar{x} \otimes \bar{x}^T B +$

$$+\bar{x} \otimes \bar{x}^T B^T, \ \ (\tilde{a}y) = (ay) + \bar{x}^T By + \bar{x}^T B^T y + \bar{x} \otimes B^T y$$

In the work (1) obviously indicated solution of tasks (1) and (2), which are used in the further.

Note that for B=0, the system (1) is called the Hess system, which was completely investigated in (2).

In these paper, we consider a problem of the following form

$$\begin{aligned} \dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + x_1(a_0x_1^2 + a_1x_1x_2 + a_2x_2^2) \\ \dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + x_2(a_0x_1^2 + a_1x_1x_2 + a_2x_2^2) \\ x(0) &= (x_{10}, x_{20})^T, \end{aligned} \tag{3}$$

where the $a_{ij}, a_0, a_1, a_2, x_{10}, x_{20}$ -real numbers.

Since the pattern of trajectories of system (3) of the eigenvalues of matrix A, and we distinguish the following cases:

1. Eigenvalues are real and different;
2. Eigenvalues are twice repeated;
3. Eigenvalues with non-zero real parts;
4. Eigenvalues are purely imaginary complex.

We proceed to the study of the above cases.

1. As it is known in (3), there exists a nonsingular matrix P, such as

$$PAP^{-1} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}.$$

First of all, we consider the case $\lambda_1 \cdot \lambda_2 \neq 0, \lambda_1 \neq \lambda_2$. Perform the following transformation of system (3)

$$y = Px$$

Then we get the canonic form (3)

$$\begin{aligned} \dot{y}_1 &= \lambda_1 y_1 + y_1(b_0y_1^2 + b_1y_1y_2 + b_2y_2^2) \\ \dot{y}_2 &= \lambda_2 y_2 + y_2(b_0y_1^2 + b_1y_1y_2 + b_2y_2^2) \\ y(0) &= (y_{10}, y_{20}). \end{aligned} \tag{4}$$

Obviously, the fundamental matrix of the linear part of the system (4) has the form

$$e^{tC} = \begin{pmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{pmatrix}.$$

Using the results of work (1), after simple calculations we obtain a solution to task (4) in the following form:

$$y_1(t) = \frac{y_{10} e^{\lambda_1 t}}{\sqrt{1 + \frac{b_0}{\lambda_1} y_{10}^2 + \frac{2b_1}{\lambda_1 + \lambda_2} y_{10} y_{20} + \frac{b_2}{\lambda_2} y_{20}^2 - B}}$$

$$y_2(t) = \frac{y_{20} e^{\lambda_2 t}}{\sqrt{1 + \frac{b_0}{\lambda_1} y_{10}^2 + \frac{2b_1}{\lambda_1 + \lambda_2} y_{10} y_{20} + \frac{b_2}{\lambda_2} y_{20}^2 - B}} \tag{5}$$

where the $B = \left(\frac{b_0}{\lambda_1} y_{10}^2 e^{2\lambda_1 t} + \frac{2b_1}{\lambda_1 + \lambda_2} y_{10} y_{20} e^{(\lambda_1 + \lambda_2)t} + \frac{b_2}{\lambda_2} y_{20}^2 e^{2\lambda_2 t} \right)$

Consider the following equations:

$$1 + \frac{b_0}{\lambda_1} y_1^2 + \frac{2b_1}{\lambda_1 + \lambda_2} y_1 y_2 + \frac{b_2}{\lambda_2} y_2^2 = 0 \tag{6}$$

In the polar coordinate system $(\rho, \varphi)$of equation (6) is written as

$$\rho = \sqrt{\frac{-2}{\frac{b_0}{\lambda_1} + \frac{b_2}{\lambda_2} - R\cos(2\varphi + \varphi_0)}}, \tag{7}$$

where the $R = \sqrt{\left(\frac{b_0}{\lambda_1} - \frac{b_2}{\lambda_2} \right)^2 + \left(\frac{b_1}{\lambda_1 + \lambda_2} \right)^2}$.

By the direct substitution, we can verify that if the initial point (y_{10}, y_{20}) satisfies equation (6), then the corresponding solution (5) also satisfies equation (6). Therefore, by the form of equations (6), (7) we can also derive the following statements:

a) If $\left| \frac{b_0}{\lambda_1} + \frac{b_2}{\lambda_2} \right| < R$ the equation (6) represents hyperbole;

b) If $\frac{b_0}{\lambda_1} + \frac{b_2}{\lambda_2} < 0$ and $R < \left| \frac{b_0}{\lambda_1} + \frac{b_2}{\lambda_2} \right|$, the equation (6) represents ellipse;

c) If $\frac{b_0}{\lambda_1} + \frac{b_2}{\lambda_2} < 0$ and $R = \left| \frac{b_0}{\lambda_1} + \frac{b_2}{\lambda_2} \right|$, the (6) represents two parallel straights;

d) If $R \leq \frac{b_0}{\lambda_1} + \frac{b_2}{\lambda_2}$, has no solution.

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Dynamics in energy models: the transportation network and its involving role

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Extended abstract [1]

In this talk we will examine the consequences of including distributed delays in an energy model. In particular, we will present a model that has been developed starting from C.L. Dalgaard and H. Strulik's model [3], a mathematical model of an economy viewed as a transport network for energy. The new model has been developed by C. Bianca et al. [1], modifying the model by C.L. Dalgaard and H. Strulik [3] with the assumption that the energy conservation formula would be influenced by a time delay; they have showed

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that the dynamics of the system is characterized by a delay differential equation. The stability behaviour of the resulting equilibrium for our dynamic system is analyzed including models with Dirac, weak and strong kernels. Applying the Hopf bifurcation theorem we will determine conditions under which limit cycle motion is born in such models. The results indicate that distributed delays have an ambivalent impact on the dynamical behaviour of systems, either stabilizing or destabilizing them. Afterwards, based on V.I. Yukalov et al. [5], C. Bianca et al. [2] have adapted their ideas and proposed a generalization by introducing a logistic-type equation for population with delayed carrying capacity. In their study they have analyzed the consequences of replacing time delays with distributed time delays. C. Bianca et al. [2] have showed that the destructive impact of the agents on the carrying capacity leads the system dynamic behaviour to exhibit stability switches and Hopf bifurcations to occur. Now we will organize a new proposal in this direction.

Keywords
Bifurcation; Energy; Network; Delayed Carrying Capacity.

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Research Dynamics in Western Balkans and the impact of EU enlargements on Science and Innovation by a Network Analysis Model

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Extended abstract

The aim of this work is to examine the publications of international co-authors belonging to some Eastern European Countries between 1994 and 2018. In this context, we want to examine whether the association with the European Union, as countries members or associates, favored Eastern European countries (referred to as East-E) by comparing all publications of these countries with those of EU members. This research question tentatively exposes the advantages in publishing under European Union schemes by the type of affiliation to the European Union itself. To do so, it identifies three subregions a priori: members of the European Union (East-EU); being an affiliated country to EU research schemes (East-AC); or neither (East-Ext). This is tested at different levels: number of publications (articles co-authored with at least one East-E presence); centrality of a given country in the global network of collaborations. The findings show that to be EU member or associated countries does play a positive role, although national differences within these different types of affiliation are more relevant than those between the three sub-regions. Findings suggest further research directed at understanding national policies concerning research, and how the European Union might consider its contribution in the wider European Research Area. These findings also suggest further research concerning the future of Eastern Europe, especially in a possible scenario of “two-speeds integration” of the European Union and the European Research Area.

Keywords
Social Network Analysis, International Collaborations, Western Balkans and EU, Substitution Effect.

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DYNAMIC SEARCH ON THE EDGES OF THE GRAPH

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In this paper the dynamic game of catching a point moving on the geometric (conjugate) graph (with rectifiable edges) in space R^d is considered. Suppose that the graph is connected and n and l are the numbers of its vertices and edges, respectively, the length of all edges equals to 1. The game involves two players moving on the edges of the graph: $\mathbf{P}$ - the team of pursuers controlling the movement of points $P_1, P_2, ..., P_m$ and Q - the evader controlling the point [1], [5], [6]. The maximum speed off all players is equal to 1.

Further, we determine the natural number $N(G)$ [2], [3], [4]. The smallest of the numbers m that $\mathbf{P}$ wins in the game is denoted by $N(G)$. If G is the free, then $N(G) = 1$, if the graph has at least one cycle, then $N(G) \geq 2$. It is obvious that $N(G)$ is exist and $N(G) \leq n - 1$.

Now, we take the point $A \in R^d$ that does not belong to the graph and connect this point with some vertex of the graph G. By connecting the point A with the vertices of the graph G, we will obtain the a geometric graph G_1 with $n + 1$ vertices and $l + 1$ edges, and having the properly $V(G) \subset V(G_1)$, $E(G) \subset E(G_1)$. Assume that the length of the edge coming out of the point A is equal to 1.

We determine the natural number $k_1 = N(G_1)$ by looking at a game involving $\mathbf{P}$ - a team of pursuers and the evader Q along the edges of the graph G_1.

Similarly, by connecting the point A with vertices of the graph G_1, we will create a geometric graph G_2 with the number of vertices $n + 1$, the number of edges $l + 2$. And, it holds: $V(G) \subset V(G_1) \subset V(G_2)$, $E(G) \subset$

$E(G_1) \subset E(G_2)$.

We determine the natural number $k_2 = N(G_2)$ by looking pursuit-evasion game on the edges of the graph G_2.

Continuing this process, we will create a geometric graph G_i with the number of vertices $n+1$, the number of edges $l+i$. And, it holds: $V(G) \subset V(G_1) \subset ... \subset V(G_i)$, $E(G) \subset E(G_1) \subset ... \subset E(G_i)$, $i = 1, 2, ..., n$.

We determine the natural number $k_i = N(G_i), i = 1, 2, ..., n$ by looking pursuit-evasion game on the edges of these graphs.

Consequently, we obtain the sequence $k_1, k_2, ..., k_n$. Denote by $K = \{k_1, k_2, ..., k_n\}$ this sequence.

Let us give some properties of K.

1. If $N(G) = 1$, then $K = \{1, 2, 2, ..., 2\}$.
2. If $N(G) > 1$, then $min\{k_1, k_2, ..., k_n\} = 2$.
3. $k_1 = N(G)$.
4. $k_n = 2$.
5. $max\{k_1, k_2, ..., k_n\} \leq k_1 + 1$.
6. $k_{i+1} - k_i \in \{-1, 0, 1\}, k_i \in K, i = 1, 2, ..., n-1$.
7. $|\{i | k_{i+1} - k_i = -1, k_i \in K, i = 1, 2, ..., n-1\}| \leq 1$, where $|\Omega|$ - the number of elements of the set Ω.
8. $|\{i | k_{i+1} - k_i = 1, k_i \in K, i = 1, 2, ..., n-1\}| = max\{k_1, k_2, ..., k_n\} - min\{k_1, k_2, ..., k_n\}$,.

Keywords

A geometric graph; Dynamic game; The team of pursuers; The evader; Pursuit-evasion game.

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PURSUIT-EVASION GAME ON THE GRAPH OF 1-SKELETON OF THE ICOSAHEDRON

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Let G denote the graph of 1-skeleton of the regular polyhedrons with icosahedron in Euclidian space R^3 [1], [3]. The team of Pursuers $\mathbf{P} = \{P_1, P_2, ..., P_m\}$ and one Evader Q moving along play a pursuit-evasion differential game. Speed of Q doesn't exceed 1 while maximal speed of point P_i equals $\rho_i, \rho_i \leq 1, i = 1, 2, ..., m$, $1 \geq \rho_1 \geq \rho_2 \geq ... \geq \rho_m > 0$. The process of pursuit-evasion begins from initial positions $\mathbf{P}(0) = \{P_1(0), P_2(0), ..., P_m(0)\}$, $Q(0)$. If one of the players chooses concrete strategy and other chooses arbitrary control function the $\mathbf{P}(t) = \{P_1(t), P_2(t), ..., P_m(t)\}, Q(t), t \geq 0$ then corresponding trajectories will be generated. The aim of the team of pursuers is to reach the equality $P_i(T) = Q(T)$ for some $i = 1, 2, ..., m$, $T \geq 0$ for any initial positions. The aim of evader is opposite, i.e. to hold the condition $P_i(t) = Q(t)$ for all $i = 1, 2, ..., m$ and $t, t \geq 0$ for some initial position (see [2]-[4]).

Obviously, if m is great enough then the team of Pursuers can win the game. The least value of m that m Pursuers win the game, will be denoted by $N(G)$.

Theorem 1. If $\rho_1 = 1, \rho_2 > 0, \rho_3 > 0$ then on the Pursuit-Evasion game, the team of Pursuers wins.

Theorem 2. If $\rho_1 = 1, \rho_2 = 10$ then on the Pursuit-Evasion game, the Evader wins.

Theorem 3. If $\rho_1 \geq \frac{2}{3}, \rho_2 \geq \frac{2}{3}, \rho_3 \geq \frac{2}{3}$ then on the Pursuit-Evasion game, the team of Pursuers wins.

Theorem 4. If $\rho_1 < 1, \rho_2 < \frac{1}{2}, \rho_3 < \frac{1}{2}$ then on the Pursuit-Evasion game, the Evader wins.

Keywords
The regular polyhedron; Icosahedron; The team of pursuers; The evader; Pursuit-evasion game.

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Pursuit differential game problem on dodecahedron

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Extended abstract [1]

The present paper is devoted to a pursuit differential game problem on 1-skeleton of dodecahedron. Let the group of three pursuers $P = \{P_1, P_2, P_3\}$ and one evader Q move along the edges (1-skeleton) of dodecahedron $A_1...A_5$-$C_1...C_{10}B_1...B_5$. Without any loss of generality we assume that the lengths of edges of dodecahedron are equal to 1. It is assumed that each player knows positions of other players at the present time t. Moreover, pursuers know the evader's velocity at the present time t as well. We use $P_i(t)$, $i = 1, 2, 3$, and $Q(t)$ to denote the positions of pursuers and evader at the time t.

It is assumed that $P_i(0) \neq Q(0)$, $i = 1, 2, 3$. We'll construct strategies for the pursuers to complete the game for any behavior of the evader. We denote the maximum speed of evader and i-th pursuer by σ and ρ_i, respectively. Without restriction of generality we assume that $\sigma = 1$.

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Definition. *If there exist strategies of pursuers such that for any control of evader $P_i(\tau) = Q(\tau)$ for some $i \in \{1, 2, 3\}$ and $\tau > 0$, then we say that the group of pursuers wins the game.*

The following statement is the main result of the paper.
Theorem. *If $\frac{2}{3} \leq \rho_1 \leq 1$, $\frac{2}{3} \leq \rho_2 \leq 1$, and $\rho_3 > 0$, then the group of pursuers P wins the game. Moreover, the game is completed by the time $\frac{25}{2\rho_3}$.*

There are many articles, devoted to pursuit and evasion differential game problems on the edge graph of polyhedron (see, for example, [1 – 4]). In the present paper, a pursuit differential game problem on the 1-skeleton of dodecahedron is studied for the first time when the speeds of pursuers are less than that of the evader.

Keywords
Pursuit differential game; evader; pursuer; dodecahedron; strategy.

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ALGORITHM OF SOLUTION OF ONE GAME OF TWO PERSONS

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Extended abstract [1]

Let $T = [0, t^*]$. The behavior of a system controlled by two players (participants), is described by the differential equation [1,2]:

$$\dot{x} = Ax + bu + dv, \quad x(0) = x_0, \tag{1}$$

where $x = x(t) = (x_1(t), x_2(t), ..., x_n(t))'$ is the state vector of system at the current time t; u and v are the control parameters of the first and second players, respectively, at the time t; A is an $n \times n$ matrix, b, d, x_0 are given n-vectors.

Piecewise constant function $u(\cdot) = (u(t), t \in T)$ being continuous from the right and satisfying the inequalities $f_* \leq u(t) \leq f^*$, $t \in T$, is called the control of the first player, where f_*, f^* are given numbers.

Impulse function [2] $v(\cdot) = (v(t), t \in T)$ with set of quantization

$$\tau = \{t_1, t_2, ..., t_l\}, \quad 0 = t_1 < t_2 < ... < t_l < t_{l+1} = t^* \quad (l \geq m),$$

satisfying the inequalities $g_*(t) \leq v(t) \leq g^*(t)$, $t \in T$, is called the control of the second player, where $g_*(t), g^*(t)$, $t \in T$, are given impulse functions with the set quantization τ, and t^* is a given positive number.

According to the theory of differential equations, for each pair $\{u(\cdot), v(\cdot)\}$ of the players' controls, there corresponds the only continuous solution $x(\cdot) = (x(t), t \in T)$ of equation (1), the trajectory of the dynamic systems.

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Let H, g, and c be given $m \times n$ constant matrix, m vector, and n vector, respectively, U, V be the sets of control functions of the first and second players, respectively. The terminal set and the objective functional are defined respectively by equations

$$M = \{x \in \mathbb{R}^n \mid Hx = g\}, \quad J(u(\cdot), v(\cdot)) = c'x(t^*).$$

Consider the following game problem. The first player chooses a control $u(\cdot) = (u(t), t \in T)$, $f_* \leq u(t) \leq f^*$, $t \in T$, and then knowing this control the second player chooses a control $v(\cdot) = (v(t), t \in T)$, $g_*(t) \leq v(t) \leq g^*(t)$, $t \in T$.

The goal of the first player is not to bring the trajectory of system (1) into the set M at the time t^* by selecting a control $u^0(\cdot) = (u^0(t), t \in T)$, if this possible, else to maximize the functional $\min_{v(\cdot)\in V} J(u(\cdot), v(\cdot))$. The goal of the second player is to bring the trajectory of system (1) from the point x_0 to the set M at t^* by choosing a control $v^0(\cdot) = (v^0(t), t \in T)$ and minimize the functional $J(u(\cdot), v(\cdot))$.

In the present paper, following [3] this problem has been investigated by using the method of special nonsmooth problem optimization.

Using the connection between these problems an algorithm has been developed for solving the stated problem. The algorithm is based on comparing the values of special controls of players in the dual problem to the special nonsmooth optimization problem.

Keywords
differential equations; a game; players; nonsmooth problem.

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Differential games of many players on the 1-skeleton of a regular simplex

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Extended abstract [1]

Pursuit and evasion differential games are studied on 1-skeleton Σ^d of the regular simplex of dimension d, which is defined as a subset of the Euclidean space $\mathbb{R}^d$ by the following relations

$$\sum z_j = \frac{1}{\sqrt{2}}, \quad z_j \geq 0, \ j = 1, 2, ..., d+1.$$

Its edges of length 1 form a complete graph Σ^d with $d+1$ vertices. Let the group of n pursuers and one evader move on Σ^d according to the following equations

$$\dot{x}_i = u_i, \qquad x_i(0) = x_{i0}, \quad i = 1, 2, ..., n, \tag{1}$$

$$\dot{y} = v, \qquad y(0) = y_0, \tag{2}$$

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where $x_i, y, x_{i0}, y_0 \in \Sigma^d$, $u_i, v \in \mathbb{R}^d$, $x_i(t)$ and $y(t)$ are the states, and u_i, v are control parameters of ith pursuer and evader, respectively. It is assumed that $|v| \leq 1$, $|u_i| \leq \rho_i$, $i = 1, 2, ..., n$, where $\rho_i < 1$, $i = 1, 2, ..., n$, are given positive numbers. It is assumed that all the players move only along the 1-skeleton of the regular simplex Σ^d.

Pursuers use information about the values $x_1(t), ..., x_n(t), y(t)$ and $v(t)$ at current time t to construct their strategies. The evader uses information about $x_1(t), ..., x_n(t), y(t)$ to construct his strategy.

Definition 1 *If there exist strategies of pursuers such that for any control of evader $x_i(\tau) = y(\tau)$ for some $i \in \{1, 2, ..., n\}$ and $\tau > 0$, then we say that pursuit can be completed in the game.*

Let $\frac{1}{2}\sigma \leq \rho_i < \sigma$, $i = 1, 2, ..., k$, and $\rho_i < \frac{1}{2}\sigma$, $i = (k+1), ..., n$ for some integer $k \geq 0$.

Theorem 2 *If either (i) $n = k$ and $n + k > d$ or (ii) $n > k$ and $n + k \geq d$, then pursuit can be completed in the game.*

Definition 3 *If there exists a strategy of evader such that for any control of pursuers $x_i(t) \neq y(t)$ for all $i = 1, 2, ..., n$, and $t > 0$, then we say that evasion is possible in the game.*

Theorem 4 *If either (i) $n = k$ and $n + k \leq d$ or (ii) $n > k$ and $n + k < d$, then evasion is possible.*

Keywords
Pursuit differential game; evasion differential game; 1-skeleton of simplex; strategy.

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The optimal control problem of an ensemble of trajectories of differential inclusion with delay under terminal constraints

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Extended abstract [1]

In the modern theory of dynamic control systems, methods of multivalued analysis and the mathematical apparatus of the theory of differential inclusions are widely used [1]. Differential inclusions with a control parameter can be used to study non-deterministic models of control systems. Methods for controlling ensembles (bundles) of trajectories of differential inclusions have effective applications in constructing problem an optimal control that gives a guaranteed value of the quality criterion [2, 3]. The problems of controlling bundles of trajectories also arise in dynamic models of conflict situations – in differential games [4].

Consider a controllable differential inclusion with delays of the form [3]

$$\frac{dx}{dt} \in A(t)x + \sum_{i=1}^{k} A_i(t)x(t-h_i) + b(t,u), \tag{1}$$

where $t \in [t_0, t_1],\ x(t) = \varphi_0(t),\ t \in [t_0 - \max\limits_{i=\overline{1,k}} h_i, t_0]$, $u \in V$.

Let: the elements of $n \times n$ -matrices $A(t)$ and $A_i(t)$, $i = \overline{1,k}$ are summable on $T = [t_0, t_1]$; multivalued mapping $(t,u) \to b(t,u) \in R^n$ with convex

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compact values, measurable by $t \in T$; support function $C(b(t,u),\psi)$ convex by $u \in V$; $\|b(t,u)\| \leq \beta(t)$, $\beta(\cdot) \in L_1(T)$; V - is a convex compact R^m; $\varphi_0(\cdot) \in C^n(T_0)$. Then: the set of admissible controls $U(T)$, consisting of all measurable functions $u = u(t) \in V, t \in T$, is weakly compact in $L_2^m(T)$; the set of absolutely continuous on T trajectories of system (1) $H(u,\varphi_0)$ is a non-empty compact set in $C^n(T_1)$; the ensemble of trajectories $t \to X(t,\, u,\varphi_0) = \{\xi \in R^n : \xi = x(t),\ x(\cdot) \in H(u,\varphi_0)\}$, $t \in T$ is a continuous multivalued mapping whose values are convex compact sets in R^n, and the support function $C(X(t,u,\varphi_0),\psi)$ is convex by $u(\cdot) \in U(T)$.

Consider the problem of terminal control of an ensemble of trajectories of system (1):

$$C(X(t_1,u,\varphi_0),l_0) \to \min,\ C(X(t_1,u,\varphi_0),l_i) \leq q_i,\ i = \overline{1,k},\ u(\cdot) \in U(T). \tag{2}$$

Definition. Problem (2) is called regular if there exists $\tilde{u}(\cdot) \in U(T)$ such that $C(X(t_1,\tilde{u},\varphi_0),l_i) < q_i$, $i = \overline{1,k}$.

Theorem. Let problem (2) is regular. Then for the optimality of $u^*(\cdot) \in U(T)$ it is necessary and sufficient the existence of $\lambda^* = (\lambda_1^*, ..., \lambda_k^*)$, $\lambda_i^* \geq 0$, $i = \overline{1,k}$ such that the pair $\{u^*(\cdot), \lambda^*\}$ constitutes the saddle point of the Lagrange functional

$$L(u(\cdot),\lambda) = C(X(t_1,u,\varphi_0),l_0) + \sum_{i=1}^{k} \lambda_i (C(X(t_1,u,\varphi_0),l_i) - q_i).$$

Necessary and sufficient optimality conditions of the following form were obtained from this theorem for the regular problem (2)

$$\min_{v \in V}[C(b(t,v),\psi^0(t)) + \sum_{i=1}^{k} C(b(t,v),\psi_i^*(t))] =$$

$$= C(b(t,u^*(t)),\psi^0(t)) + \sum_{i=1}^{k} C(b(t,u^*(t)),\psi_i^*(t)), t \in T, \tag{3}$$

where $\psi^0(t) = \psi(t,l_0)$, $\psi_i^*(t) = \psi(t,\lambda_i^* l_i)$, function $\psi(t,l)$ is the solution of the system:

$$\dot{\psi} = -A'(t)\psi(t) - \sum_{i=1}^{k} A_i'(t+h_i)\psi(t+h_i),\ \psi(t_1) = l. \tag{4}$$

It follows from the obtained result that in order to construct optimal control in the considered problem, one should first find a solution to the system (4) and then solve a parameterized finite-dimensional optimization problem (3).

Keywords
differential inclusion; ensemble of trajectories; optimal control; terminal control.

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Pursuit and Evasion Differential Games for an Infinite System of 2-systems of Differential Equations

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Extended abstract [1]

We study a differential game for the following system

$$\begin{aligned} \dot{x}_i &= -\alpha_i x_i - \beta_i y_i + u_{i1} - v_{i1}, \qquad & x_i(0) = x_{i0}, \\ \dot{y}_i &= \beta_i x_i - \alpha_i y_i + u_{i2} - v_{i2}, \qquad & y_i(0) = y_{i0}, \end{aligned} \tag{1}$$

in Hilbert space l_2, where α_i, β_i are real numbers, $\alpha_i \geq 0$, $(x_{10}, x_{20}, \ldots)$, $(y_{10}, y_{20}, \ldots) \in l_2$, $u = (u_1, u_2, ...)$ with $u_i = (u_{i1}, u_{i2})$ and $v = (v_1, v_2, ...)$ with $v_i = (v_{i1}, v_{i2})$, $i = 1, 2, ...$, are pursuer's and evader's control parameters. We assume that $0 \leq t \leq T$, where T is a sufficiently large number, and $z_0 = (x_{10}, y_{10}, x_{20}, y_{20}, \ldots) \neq 0$.

Let ρ and σ be given positive numbers. An admissible control of pursuer (evader) is a function $u(t) = (u_1(t), u_2(t), \ldots)$ $(v(t) = (v_1(t), v_2(t), \ldots))$, $t \in [0, T]$, whose coordinates $u_i(t)$ $(v_i(t))$ are measurable and satisfy the condition

$$\sum_{i=1}^{\infty} (u_{i1}^2(t) + u_{i2}^2(t)) \leq \rho^2, \quad \left(\sum_{i=1}^{\infty} (v_{i1}^2(t) + v_{i2}^2(t)) \leq \sigma^2 \right), \; 0 \leq t \leq T.$$

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Let $\rho > \sigma$, $z_i(t) = (x_i(t), y_i(t))$, $z_{i0} = (x_{i0}, y_{i0})$. We call a number θ a guaranteed pursuit time if for some strategy of pursuer and for any admissible control of the evader, $z(t') = 0$ at some t', $0 \le t' \le \theta$, where $z(t) = (z_1(t), z_2(t), \dots)$ is the solution of the initial value problem (1). The pursuer is interested in minimizing the guaranteed pursuit time.

A number τ is called a guaranteed evasion time if for any number τ', $0 \le \tau' < \tau$, we can construct an admissible control for the evader such that for any admissible control of the pursuer, we have $z(t) \neq 0$ for all $0 \le t \le \tau'$ and $i = 1, 2, \dots$. The evader is interested in minimizing the guaranteed evasion time. Problem is to find an equation for a guaranteed pursuit time and a guaranteed evasion time in the game (1).

Theorem 1. For the initial state $z_0 = (z_{10}, z_{20}, \cdots)$, the number θ that satisfy the equation

$$\sum_{\alpha_i > 0} \frac{\alpha_i^2 |z_{i0}|^2}{\sinh^2(\alpha_i \theta)} + \frac{1}{\theta^2} \sum_{\alpha_i = 0} |z_{i0}|^2 = (\rho - \sigma)^2$$

is a guaranteed pursuit time in the game (1).

Theorem 2. For the initial state $z_0 = (z_{10}, z_{20}, \cdots)$, the number

$$\tau = \sup_i \tau_i, \quad \tau_i = \begin{cases} \frac{1}{\alpha_i} \ln\left(\frac{\alpha_i |z_{i0}|}{\rho - \sigma} + 1\right), & \alpha_i > 0 \\ \frac{|z_{i0}|}{\rho - \sigma}, & \alpha_i = 0 \end{cases}$$

is a guaranteed evasion time in game (1).

Keywords
Differential game, pursuit, control, strategy, infinite system of differential equations, geometric constraint.

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Optimal Lockdown Strategies driven by Socioeconomic Costs

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Extended abstract [1]

The recent pandemic of COVID-19 that spread all over the world has put Governments at hard testing, because they have to manage a global health crisis with dramatic effects on both human lives and economies. In this research, we modify a classical SIR model to better adapt to the dynamics

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of the COVID-19. We propose the heterogeneous SQAIRD model where COVID-19 spreads over a population of economic agents, namely: the elderly, adults and young people. Within each demographic group, the population is divided in five subgroups: Susceptible (S), Quarantined (Q), Asymptomatic (A), Infected (I), Recovered (R), and Dead (D). The group of Susceptibles are those who are exposed to the virus. The group indicated by Q is the group Quarantined (i.e. isolated at home), by law or by their will. The group Asymptomatic is a group of people who caught the virus, but did not show evident symptoms. These people can behave like a susceptible but they can infect other people as well. Then they can either recover (then shift into the group of Recovered) or develop symptoms, then shifting in the group of Infected with symptoms. An Infected, instead, can either recover and shift into Recovered group or die (then shift into the Dead group). Once a person shift into the Recovered or Dead groups, it cannot be infected anymore (we assume that a person that recovers, develops immunity and cannot be reinfected). We design and simulate an optimal control problem faced by a Government, where its objective is to minimize the costs generated by the pandemics using as control a compulsory quarantine measure (that is, a lockdown). We first analyze the problem from a theoretical perspective, claiming that different lockdown policies (total lockdown, no lockdown or partial lockdown) may justified by different cost structures of the economies. We analyze a particular cost structure (convex costs) and simulate a targeted optimal policy vs. a uniform optimal policy, by dividing the whole population in three demographic groups (young, adults and old). We also simulate the dynamic of the pandemic with no policy implemented.

Keywords
Epidemic process; SIR model; Quarantine; Optimal control.

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Effectiveness of protection measures in clustered epidemic model with intralevel migration

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Extended abstract [1]

Nowadays, epidemic modelling provides an appropriate tool for describing the propagation of biological viruses in human or animal populations, information in social networks and malicious software in computer or ad hoc networks. The current study represents a hierarchical epidemic model that describes the propagation of a pathogen in the clustered human population. Estimation of Novel coronavirus spreading worldwide leads to the idea

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of the hierarchical structure of the epidemic process. Thus, the propagation process is divided into several clusters. On each cluster, the pathogen propagation process is based on the Susceptible-Exposed-Infected-Recovered (SEIR) model. We formulate the modified model of transmission of the infected individuals between the clusters. The control of pathogen spreading can be seen as an optimal control problem where a tradeoff exists between the cost of active virus propagation and the design of the appropriate quarantine or pharmaceutical measures. Its network defines each cluster in the hierarchical system.

We estimate the effectiveness of protection measures within clusters and between clusters of the population. Intralevel control is defined by increasing the proportion of the population in a Quarantine and increasing the effectiveness of the treatment of infected agents. In contrast, inter-level control impacts the intensity of migration rate between clusters. Thus, we compare pharmaceutical interventions with several types of non-pharmaceutical ones. By series of numerical experiments, we demonstrate the network structure's influence on the interaction between clusters and inside clusters in a hierarchical epidemic model. The series of numerical experiments are corroborated the obtained results.

Keywords
Epidemic process; SIR model; Quarantine; Optimal control.

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Differential Games with Grönwall Type Constraints on Controls

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Extended abstract [1]

In the present work, a simple motion pursuit-evasion differential game of one pursuer and one evader is studied. The strategies of players are constructed and solvability conditions of the pursuit-evasion game are given.

We propose a new set of controls of pursuer and evader described by generalized Grönwall type constraints

$$|u(t)| \leq \rho_0 + \rho_1 t + k \int\limits_0^t |u(s)|ds, \text{ a.e. } t \geq 0,$$

$$|v(t)| \leq \sigma_0 + \sigma_1 t + k \int\limits_0^t |v(s)|ds, \text{ a.e. } t \geq 0,$$

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respectively, where $\rho_0>0$, $\sigma_0 > 0$, $\rho_1 > 0$, $\sigma_1 > 0$, $k \geq 0$.

Let dynamics of pursuer P and evader E be described by the equations

$$P\text{:}\ \ \dot{x} = u, \quad x(0) = x_0,$$

$$E\text{:}\ \ \dot{y} = v, \quad y(0) = y_0,$$

where $x, y, x_0, y_0, u, v \in \mathbb{R}^n, n \geq 1, x_0 \neq y_0$.

The aim of the pursuer is capture, i.e., to reach $x(t) = y(t)$ and the evader struggles to avoid an encounter, i.e., to achieve $x(t) \neq y(t)$ for all $t \geq 0$, and in the opposite case, to postpone the instant of encounter as long as possible.

Definition 1. If $\delta_0 \geq 0$, $\delta_1 \geq 0$, then the function

$$\mathbf{u}_{Gr}(t, v) = v - \lambda_{Gr}(t, v)\xi_0 \tag{1}$$

is called a Π_{Gr}-strategy of the pursuer in the pursuit game, where $\lambda_{Gr}(t, v) = \langle v, \xi_0\rangle + \sqrt{\langle v, \xi_0\rangle^2 + \varphi^2(t) - |v|^2}$, $\xi_0 = z_0/|z_0|$, $\delta_0 = \rho_0 - \sigma_0$, $\delta_1 = \rho_1 - \sigma_1$, $\varphi(t) = \frac{\rho_1}{k}(e^{kt} - 1) + \rho_0 e^{kt}$, $\varphi(0) = \rho_0$.

Proposition 1. Assume that $\delta_0 \geq 0$, $\delta_1 > 0$ or $\delta_0 > 0$, $\delta_1 \geq 0$. Then there exists at least one positive root of the equation

$$e^{kt} = At + B \tag{2}$$

with respect to t, where $A = \frac{k\delta_1}{\delta_1 + k\delta_0}$, $B = 1 + \frac{k^2|z_0|}{\delta_1 + k\delta_0}$. The smallest root of equation (2) is called call a guaranteed pursuit time and denoted by T_{Gr}.

Theorem 1. Let Proposition 1 is valid. Then the strategy (1) guarantees completion of pursuit on the time interval $[0, T_{Gr}]$.

To solve the evasion problem we will propose a strategy of the evader.

Definition 2. We call a strategy of the evader control function

$$\mathbf{v}_{Gr}(t) = -\psi(t)\xi_0, \ t \geq 0. \tag{3}$$

where $\psi(t) = \frac{\sigma_1}{k}(e^{kt} - 1) + \sigma_0 e^{kt}$, $\psi(0) = \sigma_0$.

Theorem 2. If $\delta_0 \leq 0$, $\delta_1 \leq 0$, then the strategy (3) is winning for the evader in the evasion game.

Keywords
Differential game; pursuit; evasion; Grönwall type constraint; strategy.

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Differential game: life line in the Pontryagin control example

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Extended abstract [1]

In this work, it is studied the problem of R.Isaacs called "Life line" game in $\mathbb{R}^n$. Here pursuit problem will be solved by parallel pursuit strategy.

Let dynamics of the players be described by the equations

$$P\text{:}\ \ \ddot{x} - a\dot{x} = u,\ x(0) = x_0,\ \dot{x}(0) = x_1,\ \ |u(t)| \le \alpha \ \text{ a.e. } t \ge 0,$$

$$E\text{:}\ \ \ddot{y} - a\dot{y} = v,\ y(0) = y_0,\ \dot{y}(0) = y_1,\ \ |v(t)| \le \beta \ \text{ a.e. } t \ge 0,$$

where $x, y, u, v \in \mathbb{R}^n,\ n \ge 2,\ a \ne 0,\ x_0 \ne y_0,\ x_1 = y_1,\ \alpha > 0,\ \beta \ge 0$.

P aims to catch E i.e. to realize $x(t) = y(t)$ for some $t > 0$, while E stays in the zone $\mathbb{R}^n \setminus M$. The aim of E is to reach the zone M before being caught by P or to keep $x(t) \ne y(t)$ for all $t, t \ge 0$. Notice that M doesn't restrict motion of P and $y_0 \notin M$.

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Let $z = x - y,\ z_0 = x_0 - y_0$.

Definition 1. If $\alpha \geq \beta$ for all $t \geq 0$, then the function

$$\mathbf{u}(z_0, v) = v - \lambda(z_0, v)\xi_0,$$

is called Π_G-strategy for P, where $\lambda(z_0, v) = \langle v, \xi_0 \rangle + \sqrt{\langle v, \xi_0 \rangle^2 + \alpha^2 - |v|^2}$, $\xi_0 = z_0/|z_0|$.

Definition 2. The smallest positive root of the equation $e^{at} = at + 1 + \frac{a^2|z_0|}{\alpha - \beta}$ is called a guaranteed pursuit time and denoted by T.

Theorem 1. a) Let $\alpha > \beta$. Then Π_G-strategy is winning for P on time interval $[0, T]$; b) If $\alpha \leq \beta$, then for any control of P, the strategy $v(t) = -\beta\xi_0$ is winning for E i.e. $|z(t)| \geq |z_0|$ on time interval $[0, \infty)$.

Let $\alpha > \beta,\ \varphi(t) = \frac{1}{a}(e^{at} - 1)$. Then for the pair $(x(t), y(t)))$ we construct the set

$$W(t) = W(x(t), y(t)) = \{w : \beta|w - x(t)| \geq \alpha|w - y(t)|\},$$

$$W(0) = W(x_0, y_0) = \{w : \beta|w - x_0| \geq \alpha|w - y_0|\}.$$

It is clear that $y(t) \in W(t)$ for all $t \in [0, T]$.

Theorem 2. The multi-valued mapping $W(t) - \varphi(t)x_1$ is monotone decreasing in t, $t \in [0, T]$, i.e. $W(t_1) - \varphi(t_1)x_1 \supset W(t_2) - \varphi(t_2)x_1$ when $0 \leq t_1 \leq t_2$ for any $t_1, t_2 \in [0, T]$.

Theorem 3. If $W_P \bigcap M = \emptyset$, then the Π_G-strategy is winning for P, where $W_P = \{W(0) + \varphi(t)x_1 : t \in [0, T]\}$.

Theorem 4. If $W_E \bigcap M \neq \emptyset$, then there exists some control of E which is winning, where

$$W_E = \left\{ \tilde{\omega} : \tilde{\omega} = \varphi(\tau)x_1 + \frac{\beta(\omega - y_0)}{a|\omega - y_0|}(\varphi(\tau) - \tau) + y_0,\ y(\tau) = \tilde{\omega},\ \omega \in W(0) \right\}.$$

Keywords

Differential game; pursuit; evasion; strategy; life line.

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From Optimal Execution to Transient Impact Model: A Theoretical Insight

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Extended abstract [1]

Markets are far from being perfectly elastic and any order or trade causes prices to move. The relation between trades and price generated by correlation in order flows is known as market impact. In the seminal work of [1] the market impact is modeled by a constant fixed over time, making market impact permanent and constant. However, empirical evidence has shown that the *transient impact model* (TIM) provides a better way to describe how past orders influence the price, [2]. The price impact may be attributed to a stylized statistical fact induced by a mechanical consequence of the order book dynamics. Nevertheless, the origin of TIM is still unclear from a theoretical point of view. Moreover, the empirical evidence suggests to relate the transient impact to optimal schedule strategies of agents, [3].

In this work we analyze the relation of TIM and the classical Almgren and Chriss framework finding a correspondence between these two models in a simple market setting from an optimal execution inverse problem perspective.

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We start by deriving the agent Nash equilibria in a suitable market impact game setting of [4] where two agents liquidate the same asset. Then, we show how to relate the corresponding Nash equilibrium to the solution of an equivalent optimal execution problem investigating how the mutual influence of traders implies the existence of an intrinsic market impact. We found that the TIM emerges as a result of the interaction between agents in a market impact game with permanent constant price impact as in the Almgren and Chriss model.

The transient impact function solves a singular linear problem and we show that a linear decay kernel solution exists. We discuss how transaction cost levels of market impact games affect agents' interaction and the related decay kernel solutions. We also study the relations accounting for cross-impact effects and many competitors are present. Finally, we conclude by presenting an alternative approach based on price dynamics that can be employed in general model settings, from which we derive a unique nonlinear solution.

Keywords
Optimal Execution; Market Impact; Transient Impact Model; Market microstructure; Game theory.

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