# Polarization and coherence in mean field games driven by private and social utility

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## Extended abstract [1]

We analyze a simple continuous-time dynamic multi-agent model and study the limit as the number of agents goes to infinity. We consider a group of N interacting agents, who aim at minimizing an individual cost, which is comprised by a quadratic running cost and a final reward. The running cost is related to the rate at which a change in the state happens: changing opinion may be a costly operation. The final reward is given as the sum of two different terms: the willingness to align with the majority (conformism) and the aspiration of sticking with the own type (stubbornness). These two terms are possibly competing and represent a classical social dilemma: conformism, i.e., the adherence to social norms, versus stubbornness, i.e., the

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aspiration of the agent to stay as close as possible to the prescription of personal traits. Our aim is to understand the system's behavior in the limit as N tends to infinity. This falls into the realm of mean field games. Depending on the parameters of the model, the game may have more than one Nash equilibrium, even though the corresponding N-player game does not. Moreover, it exhibits a very rich phase diagram, where a number of different types of equilibria can be identified: polarized/unpolarized (related to the size of the majority), coherent/incoherent (alignment of the final population state with the initial state). These different equilibria may coexist, except for T small, where the equilibrium is always unique. We fully describe such phase diagram in closed form and provide a detailed numerical analysis of the N-player counterpart of the mean field game. In this finite dimensional setting, the equilibrium selected by the population of players is always coherent (favoring the subpopulation whose type is aligned with the initial condition), but it does not necessarily minimize the cost functional. Rather, it seems that, among the coherent ones, the equilibrium prevailing is the one that most benefits the underdog subpopulation forced to change opinion.

**Keywords**
Finite Population Dynamics; Mean field games; Multiple Nash Equilibria; Phase transition; Social Interaction.

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**Session:** Interactions and complexity in social dynamics
**Organizers:** Rosario Maggistro, Marco Tolotti

# Dangerous tangents: an application of $\Gamma$-convergence to the control of dynamical systems

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## Extended abstract [1]

Understanding and controlling collective behavior is a challenging and potentially very useful endeavour. The mere attempt to know agents' preferences and motives is fraught with practical and conceptual difficulties. Moreover, even when a reasonable model for individual actions is agreed, results can be puzzling as shown, for instance, in Schelling's segregation model [5] where slight homophilic preferences for neighbors of the same race lead in equilibrium to massive residential segregation. These collective models are known to produce very diverse outcomes, therefore, a policy maker could be interested in controlling the behavior of the population by setting, in her capacity, some

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parameters at some optimal level. In this paper, inspired by the classical riot model proposed by Granovetter [4], we consider a parametric stochastic dynamical system describing the collective behavior of a large population of interacting agents. Moreover, by controlling a parameter, a policy maker aims at maximizing her own utility which, in turn, depends on the steady state of the system. We show that this economically sensible optimization is ill-posed and illustrate a novel way to tackle this practical and formal issue. First, we introduce a mean-regularized stochastic version of the original problem with a finite number $N$ of agents. Albeit stochastic, this problem is well-posed and admits a unique maximizer for every $N$. Then, by using $\Gamma$-convergence, we are able to provide a formal limit for $N \to +\infty$ of the sequence of the mean-regularized objective functions, and to show that the corresponding maximizers converge towards a unique value which intuitively is the solution of the original ill-posed problem.
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**Keywords**
Dynamical Systems; Finite Population Dynamics; $\Gamma$-convergence; Saddle-node Bifurcations; Social Interaction.

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# Mobility Choices and Strategic Interactions in a Two-Group Macroeconomic-Epidemiological Model

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## Extended abstract [1]

We analyze the implications of strategic interactions between two heterogenous groups (i.e., young and old, men and women) in a macroeconomic-epidemiological framework. The interactions between groups determine the overall prevalence of a communicable disease, which in turn affects the level of economic activity. Individuals may lower their disease exposure by reducing their mobility, but since changing mobility patterns is costly each group

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has an incentive to free ride negatively affecting the chances of disease containment at the aggregate level. By focusing on an early epidemic setting, we explicitly characterize the cooperative and noncooperative equilibria, determining how the inefficiency induced by noncooperation (i.e., failure to internalize epidemic extenalities) depends both on economic and epidemiological parameters. We show that long run eradication may be possible even in the absence of coordination, but coordination leads to a faster reduction in the number of infectives in finite time. Moreover, the inefficiency induced by noncooperation increases (decreases) with the factors increasing (decreasing) the pace of the disease spread.
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**Keywords**
Dynamic Programming; Infectious Diseases; Macroeconomic Outcomes; Mobility Choices; Noncooperative vs Cooperative Games.

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# Infectious Diseases, Social Norms and Vaccination Decisions: Rational Opposition and Free-Riding Effects

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## Extended abstract [1]

Growing opposition to vaccination and free-riding opportunities are undermining the success of national immunization programs giving rise to disease outbreaks, as recently happened in the case of measles in Europe. This paper analyzes how individuals, by making their vaccination decisions by comparing the benefits and costs of vaccination accounting for social and health factors, determine the dynamic evolution of the overall immunization coverage and thus of disease prevalence. We show that parental decisions about whether vaccinating their children critically affect whether the disease will naturally tend to die out or to persist in the long run, and that the convergence to the former or the latter scenario depends on how some key socio-health-economic parameters (i.e., the vaccine confidence level, the degree of conformism to social norms, and the degree of disease concern) relate to key epidemiological parameters (i.e., the basic reproduction number). According to different parameter configurations, a variety of alternative outcomes may arise, such as unique and multiple equilibria, monotonic and fluctuating trajectories, chaotic dynamics. By comparing the behavioral

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and epidemiological outcomes in the stochastic framework and in its deterministic approximation, we show that the deterministic setting may lead to misleading conclusions about the true stochastic dynamics because of the metastable properties of the stochastic system in the presence of multiple stable equilibria. Public policy by affecting the key socio-health-economic parameters may play a fundamental role in order to rule out some undesired outcomes and ensure long run disease eradication. We present a calibration based on the recent measles outbreak in Italy, showing that our setup is able to well describe real world behavioral and epidemiological outcomes.

To be presented at the stream: "Interactions and complexity in social dynamics".

**Keywords**
Immunization Coverage; Infectious Diseases; Social Interactions; Social Norms; Vaccination Decisions