ASSOCIAZIONE PER LA MATEMATICA APPLICATA ALLE SCIENZE ECONOMICHE E SOCIALI

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45th AMASES MEETING

GAMES SESSIONS

September 16th (9.30 – 17.15) - ZOOM

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The Games session is divided in four parts (each presentation has 20 min. + 5 min. for Q&A): GTM ROME

DG1	Dynamic Games 1: Chair Alessandra Buratto	Authors	
36	An evasion problem for differential game with coordinatewise integral constraints	Bruno Antonio Pansera and Massimiliano Ferrara	9.30
99	New characteristic function for two stage games with spanning tree	Min Cheng, Yin Li and Ovanes Petrosian	9.55
120	Improving the estimate for the pursuer's distance from the origin	Risman Mat Hasim, Gafurjan Ibragimov, Idham Arif Alias, Guljamol Kholmirzaeva and Miraziz Makhmudov	10.20
	Differential games of the second order with integral constraints	Inomiddinov Sardorbek Nizomiddinovich	10.45
	Coffee break	11.10 – 11.30	
NCG	Non Cooperative Games: Chair Laura Ziani	Authors	
16	A McKean-Vlasov Game of Commodity Production, Consumption and Trading	Ofelia Bonesini, René Aïd, Giorgia Callegaro and Luciano Campi	11.30
56	Psychological Nash Equilibria under Ambiguity	Giuseppe De Marco, Maria Romaniello and Alba Roviello	11.55
122	Stackelberg-Nash equilibrium and quasi harmonic games	Lina Mallozzi and Armando Sacco	12.20
	Lunch break	12.45 – 14.00	
DG2	Dynamic Games 2: Chair Alessandra Buratto	Authors	
9	An Option Games Approach to valuate urban redevelopment projects achieving by Public-Private Partnership	Antonio Di Bari and Giovanni Villani	14.00
151	Bayesian Dynamic Updating in Dynamic Game Model	Jiangjing Zhou, Ovanes Petrosian and Hongwei Gao	14.25
84	Social Norms for the stability of International Environmental Agreements	Marta Biancardi, Lucia Maddalena and Giovanni Villani	14.50
98	A differential game model for an efficient pro-vax advertising campaign	Alessandra Buratto, Rudy Cesaretto and Maddalena Muttoni	15.15
	Coffee break	15.40 – 16.00	
CG	Cooperative Games: Chair Michela Chessa	Authors	
38	Minimal winning coalitions and orders of criticality	Michele Aleandri, Marco Dall'Aglio, Vito Fragnelli and Stefano Moretti	16.00
110	Dynamic Shapley value for 2-Stage cost-sharing game with arborescences	Peichen Ye, Yin Li and Ovanes Petrosian	16.25
127	Demand commitment bargaining and bounded rationality: An experimental analysis	Michela Chessa, Nobuyuki Hanaki, Aymeric Lardon and Takashi Yamada	16.50

AN EVASION PROBLEM FOR DIFFERENTIAL GAME WITH COORDINATEWISE INTEGRAL CONSTRAINTS

Massimiliano Ferrara
Department of Law, Economics and Human Sciences & Decisions_ Lab,
University *Mediterranea* of Reggio Calabria
&
ICRIOS, Department of Management and Technology
Bocconi University, Milano
massimiliano.ferrara@unirc.it

Bruno Antonio Pansera
Department of Law, Economics and Human Sciences & Decisions_ Lab,
University *Mediterranea* of Reggio Calabria
bruno.pansera@unirc.it

Extended abstract [1]

A differential game of many pursuers and one evader is described by m linear differential equations of variables x_1, x_2,..., x_m. The control parameters of players are subjected to coordinatewise integral constraints. If at least for one $i \in \{1, 2, ..., m\}$, we have $x_i(t) \neq 0$ for all $t \geq 0$, then we say that evasion is possible. The problem is to find a condition of evasion in the game. A sufficient condition of evasion is proposed in terms of energies of players. Also, a strategy for the evader that guarantees the evasion is constructed.

Keywords
Differential game; Pursuit- evation game; Strategy; Evasion.

[1] Speaker:Bruno Antonio Pansera, bruno.pansera@unirc.it.

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New characteristic function for two stage games with spanning tree

Min Cheng
Faculty of Applied Mathematics and Control Processes,
St.Petersburg State University,
cm014033@163.com

Yin Li
Faculty of Applied Mathematics and Control Processes,
St.Petersburg State University,
liyinrus@outlook.com

Ovanes Petrosian
Faculty of Applied Mathematics and Control Processes,
St.Petersburg State University,
petrosian.ovanes@yandex.ru

Extended abstract [1]

Two-stage n-player game with spanning tree is considered. The cooperative behaviour of players is defined. After the first stage game, a specified player leaves the game with a probability that depends on actions of all players in the first stage. A new approach to the construction of the characteristic function is proposed. In the game, assume that all players in coalition $S \subset N$ want to connect to the source, the coalition $N \setminus S$ has already connected to the source, and the coalition S may connect to the source through $N \setminus S$. With the help of this new characteristic function, the dynamic Shapley value is constructed. Some properties of the new characteristic function in the game are given.

Keywords
Dynamic game; Minimum cost spanning tree; Dynamic Shapley value.

[1] Speaker: Min Cheng, cm014033@163.com

Improving the estimate for the pursuer's distance from the origin.

Risman Mat Hasim
Department of Mathematics and Statistics, University Putra Malaysia
risman@upm.edu.my

Gafurjan Ibragimov
Department of Mathematics and Statistics, University Putra Malaysia
ibragimov@upm.edu.my

Idham Arif Alias
Department of Mathematics and Statistics, University Putra Malaysia
idham_aa@upm.edu.my

Guljamol Kholmirzaeva
Andijan State University, Andijan, Uzbekistan
Xolmirzayeva1991@mail.ru

Miraziz Makhmudov
Retraining and in-service institute of managers and specialists of system of public education named after A. Avloni, Tashkent, Uzbekistan
mirazizmakhmudov01@gmail.com

Extended abstract [1]

We study a pursuit differential game problem with simple motion dynamics of players in $\mathbb{R}^n$. One pursuer and one evader move in a given bounded closed convex subset of $\mathbb{R}^n$. The dynamic possibilities of players are equal. If the pursuer becomes in l vicinity of the evader, then pursuit is considered to be completed, where l is a given positive number. This game relates to lion and man games. In the past, various solutions of the game problem have been given by many researchers. In this paper, the estimate obtained by previous researchers for the distance of pursuer from the origin is improved.

[1] Speaker: Risman Mat Hasim, risman@upm.edu.my

Note that in the game analyzed in [5], time is discrete. First the man M moves from his initial state M_0 to a point M_1 for which $M_0M_1 \leq 1$, then the lion L moves from his initial state L_0 to a point L_1 for which $L_0L_1 \leq 1$ and so on. If $L_k = M_k$ for some step k, then lion wins. If $L_k \neq M_k$ for all $k = 1, 2, ...$, then man wins. In that paper, the lion L applies the discrete analogue of Rado's strategy with respect to a center C, and the lion moves from L to L' with $LL' = 1$, then it is proved that $CL'^2 \geq CL^2 + 1$ (Lemma 1, [5]). In the present paper, our goal is to improve this inequality.

Take the point x_0 as the origin, i.e $x_0 = 0$. First, we construct a piecewise constant strategy for the pursuer. Define $t_i = i\varepsilon, i = 0, 1, 2, ...$, where $\varepsilon = \frac{l}{2}$. Let the evader apply an arbitrary control $v(t)$, $t \geq 0$, and $y(t)$ be state of evader.

Theorem

For any integer $n \geq 1$, the inequality

$$|x_{n+1}| > \sqrt{|x_n|^2 + 2\varepsilon^2}$$

holds.

Keywords
Differential game; Pursuer; Evader; State constraint; Strategy; Control.

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DIFFERENTIAL GAMES OF THE SECOND ORDER WITH INTEGRAL CONSTRAINTS

Inomiddinov Sardorbek Nizomiddinovich

Namangan State University

s.inomiddinov@umail.uz

Keywords: integral constraints, differential game, pursuer, evader, acceleration.

Consider two players, a pursuer and an evader, moving in the space $\boldsymbol{R^n}$. The subscripts **P** and **E** will be reserved for the Pursuer and Evader, respectively. Suppose x and y are the locations of the pursuer and the evader respectively.

Let objects P and E with opposite aim be given in the space R^n and their movements based on the following differential equations and initial conditions:

$$\mathbf{P}:\ \ddot{x} = u, \qquad x_1 - kx_0 = 0, \tag{1}$$

$$\mathbf{E}:\ \ddot{y} = v, \qquad y_1 - ky_0 = 0, \tag{2}$$

and assume that $x_0 \neq y_0$

where $x, y, u, v \in \boldsymbol{R^n}$, $n \geq 1$ and k is arbitrary number; $x_0 = x(0), y_0 = y(0)$ are initial states of the players **P** and **E** correspondingly at $t = 0;$ $x_1 = \dot{x}(0)$, $y_1 = \dot{y}(0)$ are initial velocity of the pursuer **P** and the evader **E** respectively at $t = 0;$ u is a control parameter (controlled acceleration) of the pursuer, v is that of the evader. Control parameters u, v are selected from the class of measurable functions U_I, V_I satisfying according to integral constraints (3) and (4) (briefly,I- constraints) respectively:

$$\int_0^t (t-s)|u(s)|^2\, ds \leq \rho_0 + \rho_1 t, \qquad \rho_1 = k\rho_0 \tag{3}$$

$$\int_0^t (t-s)|v(s)|^2\, ds \leq \sigma_0 + \sigma_1 t, \qquad \sigma_1 = k\sigma_0 \tag{4}$$

and assume that $\rho_1 = k\rho_0$, $\sigma_1 = k\sigma_0$; where ρ_0, ρ_1, σ_0, σ_1 are given positive numbers[1].

The measurable functions $u(\cdot) \in U_I$ and $v(\cdot) \in V_I$ are called admissible controls of the pursuer **P** and the evader **E** respectively.

Once the players admissible controls $u(\cdot)$ and $v(\cdot)$ are chosen, the corresponding motions $x(\cdot)$ and $y(\cdot)$ of the players are defined as

$$x(t) = x_0 + tx_1 + \int_0^t (t-s)u(s)ds, \quad y(t) = y_0 + ty_1 + \int_0^t (t-s)v(s)ds$$

The object of the pursuer **P** is capture, i.e., to reach the equality

$$x(t^*) = y(t^*) \qquad (5)$$

and the evader **E** struggles to avoid an encounter, i.e., to achieve the inequality $x(t) \neq y(t)$ for all $t \geq 0$, and in the opposite case, to postpone the instant of encounter (5) as long as possible.

Definition 1. For the differential game (1)-(4), time t^* is called a guaranteed pursuit time if it is equal to an upper boundary of all the finite values of pursuit time satisfying the equality (5)[2].

To construct a strategy for the pursuer, first we assume that pursuer knows $t, x(t), y(t), v(t)$ at the current time t. Let $z(t) = x(t) - y(t)$. Now, we construct the strategy of the pursuer for the position $(t^*, z(t^*), \rho(t^*), \sigma(t^*))$ such that $z(t^*) = 0$, $\rho(t^*) = \sigma(t^*)$ [4].

We assume that at some t^* time the equality $z(t^*) = 0$ occurred, At this time

$$z(t^*) = 0 \Leftrightarrow x(t^*) = y(t^*)$$

Defenition 2. The function

$$u(v) = v - \lambda(v)\xi_0 \quad (6)$$

is called the strategy of parallel pursuit (the Π-strategy) of the pursuer **P** in the game (1) - (4), where $\lambda(v) = \max\{0, \bar{\delta} + 2\langle v, \xi_0 \rangle\}$, $\bar{\delta} = \delta_0 / |z_0|$, $\delta_0 = \rho_0 - \sigma_0$, $\xi_0 = z_0 / |z_0|$, $\langle v, \xi_0 \rangle$ – the scalar production in $\boldsymbol{R}^n$ [3].

Theorem. Let one of the following conditions holds:

1. $\rho_0 > \sigma_0$ and $k = 0$; or 2. $\rho_0 = \sigma_0$ and $k < 0$.

Then by virtue of strategy (6) the guaranteed pursuit time will be as follows:

$$t^* = \begin{cases} \sqrt{2}|z_0| / \left(\sqrt{\rho_0} - \sqrt{\sigma_0}\right), & if \quad \rho_0 > \sigma_0,\ k = 0, \\ -1/k, & if \quad \rho_0 = \sigma_0,\ k < 0, \\ positive \ \ root & if \quad \rho_0 > \sigma_0 \ \ k > 0. \end{cases}$$

Conclusion. We have studied pursuit differential games of the second order with integral constraints on control function of the players. We constructed the Π-strategy of the pursuer and gave formulae to find the guaranteed pursuit time.

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A McKean-Vlasov Game of Commodity Production, Consumption and Trading

René Aïd
Department of Economics,
Université Paris-Dauphine, PSL Research University
rene.aid@dauphine.psl.eu

Ofelia Bonesini
Department of Mathematics,
University of Padova,
bonesini@math.unipd.it

Giorgia Callegaro
Department of Mathematics,
University of Padova,
gcallega@math.unipd.it

Luciano Campi
Department of Mathematics,
University of Milan,
luciano.campi@unimi.it

Extended abstract [1]

We consider a model in which a producer affects the price dynamics of the good controlling drift and volatility of her production rate, while a consumer manipulates the price dynamics through his consumption rate in a similar way. We assume that the producer has a short position in a forward contract on λ units of the underlying at a fixed price K, while the consumer has the corresponding long position. Moreover, both players are risk averse with respect to their financial position and their risk aversion is modelled through an integrated-variance penalization. We study the impact of risk-aversion on the interaction between the producer and the consumer

[1] Speaker: Ofelia Bonesini, bonesini@math.unipd.it.

as well as on the derivative price. Mathematically speaking we are dealing with a two-player Linear-Quadratic McKean-Vlasov Stochastic Differential Game. Using methods based on BSDEs, we find Nash-equilibrium strategies in semi-explicit form. After solving for an equilibrium, we compute the two indifference prices (one for the producer and one for the consumer) induced by the equilibrium strategies of the players. We look for a quantity λ such that the players agree on the price. We illustrate our results with numerics. In particular, we investigate how risk aversion of producer (resp. consumer) affects the price that in our setting is defined as agreement indifference price, looking for some contango or backwardation effects.

Keywords
Indifference Pricing, Mean-Field SDEs, McKean-Vlasov, BSDEs.

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Psychological Nash Equilibria under Ambiguity

Giuseppe De Marco
Department of Management and Quantitative Sciences,
University of Naples Parthenope.
Center for Studies in Economics and Finance,
University of Naples Federico II.
giuseppe.demarco@uniparthenope.it

Maria Romaniello
Department of Economics,
University of Campania Luigi Vanvitelli.
maria.romaniello@unicampania.it

Alba Roviello
Department of Economics,
University of Campania Luigi Vanvitelli.
alba.roviello@unicampania.it

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Extended abstract[1]

Psychological games have been introduced to understand how emotions and opinions of players can affect a game ([1], [2]). In the paper [3] payoffs are assumed to be dependent not only on the strategies, but also on the beliefs of each player: players may have belief-dependent motivations or may believe that their opponents have belief-dependent motivations. In [3] it is presented and proved the existence of an equilibrium concept for this class of games based on the idea that the entire hierarchy of beliefs of each player must be correct in equilibrium.

[1]Speaker: Alba Roviello, alba.roviello@unicampania.it

There is another strand of literature that focuses on strategic ambiguity in classical games as it is well known that players may have ambiguous beliefs about opponents' strategy choices ([4]). In addition, limit results containing the assumptions which ensure the convergence of sequences of equilibria of ambiguous games to equilibria of the unperturbed ones can be found ([5]).

In this paper, we consider the issue of strategic ambiguity in the framework of psychological games by taking into account ambiguous hierarchies of beliefs. In particular, we focus on the effects of ambiguity on the existence of psychological Nash equilibria. We describe the model of psychological games under ambiguity and provide a generalization of psychological equilibrium concept. Numerical examples illustrating the differences between the ambiguous setting and the classical one are also shown. Then, we prove the existence of psychological Nash equilibria under ambiguity, focusing on maxmin preferences. Finally, we give a general limit theorem for sequences of equilibria of perturbed psychological games under ambiguity and we apply it to obtain a selection criterion for classical psychological equilibria, based on stability with respect to ambiguous trembles of beliefs.

Keywords: Ambiguity; Psychological games; Nash equilibria; Stability.

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Stackelberg-Nash equilibrium and quasi harmonic games

Lina Mallozzi
Department of Mathematics and Applications,
University of Naples Federico II,
Via Claudio 21, 80125 Naples, Italy
mallozzi@unina.it

Armando Sacco
Department of Management and Quantitative Studies,
Parthenope University of Naples,
Via Generale Parisi 13, 80132 Naples, Italy
armando.sacco@uniparthenope.it

Extended abstract [1]

This paper consider a two stage hierarchical game, with one leader and two followers. In the first stage (upper level), the leader announces is strategy, while in the second stage (lower level) the followers react to the leader playing a non cooperative game between them. The strategy space of the leader is a continuum, while strategy spaces of the followers are finite sets. In the case of multiple equilibria in the second stage, the leader has to face the problem that he can not be sure about followers' reaction. One possibility in this case, is to consider the so-called *security strategy*: the leader, assuming a pessimistic point of view, will find his optimal strategy by considering the worst case scenario about the Nash equilibrium in the second stage. One problem is that the non cooperative game in the lower level may have no Nash equilibrium solution. A classical example is the class of harmonic games introduced in [1]. Consider the set of mixed strategies for the followers is one way to solve this problem. But, when the leader faces a continuous strategy space, a security strategy may not exists even if: i) the leader has a nice cost function on compact set, ii) the followers play a bi-matrix game, iii) mixed

[1]Speaker: Armando Sacco, armando.sacco@uniparthenope.it.

strategies are considered in the lower level. This problem has been studied in [2] and [3]. Then, in our model an approximate solution, called ε-*approximate mixed security strategy* ($\varepsilon > 0$), is considered, and sufficient conditions on the data for its existence are proved, together with the convergence of the corresponding values as $\varepsilon \to 0^+$. Moreover, the same conditions guarantee the lower semi-continuity of the best reply correspondences also. To end, a new class of games, called *quasi harmonic games* is introduced. For this class is it possible to solve the hierarchical problem and to provide an approximate security solution under mild assumption on the data.

Keywords
Security solutions; Mixed Nash equilibrium; Quasi harmonic games.

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An Option Games Approach to valuate urban redevelopment projects achieving by Public-Private Partnership

Antonio Di Bari
Department of Economics and Finance,
University of Bari, Largo Abbazia S. Scolastica, 53 - 70124 - Bari, Italy
antonio.dibari@uniba.it

Giovanni Villani
Department of Economics and Finance,
University of Bari, Largo Abbazia S. Scolastica, 53 - 70124 - Bari, Italy
giovanni.villani@uniba.it

Extended abstract [1]

This article proposes a reliable methodology to valuate Public-Private Partnership (PPP) investment in urban building redevelopment. Since public institution could need private firms support to achieve public projects such as redevelopment investments, PPP system is often involved. This happens when the concessionaire (consortium of private firms) brings fund and management know-how to built an infrastructure and then it recovers costs using revenues generated by infrastructure assets (see [1]). In our specific case, public and private sector decide to transform out-dated building in an infrastructure that could be able to generate cash inflows.

The valuation of redevelopment investments is not a simple task since they

[1] Speaker:[Antonio Di Bari antonio.dibari@uniba.it].

are characterized by sequential nature and uncertainty. Regarding the first feature, we can state that the redevelopment process is characterized by sequential stages that require different amount of expected costs related to each stage to implement them. The insight on the basis of these projects is that private firms choose to invest in the future stage only if the performance of investment evolution in the previous stage is successful. Previous studies faced this type of investments using compound Real Option Approach (ROA) (see [3]). Moreover, these investments are also characterized by various risks that imply their uncertain nature (see [4]). Among them, there is the competition risk that leads the choice of investment to depend also on the fact that new competitor can enter the market. This is the case of strategic interactions captured by Game Theory (GT).

In this paper we combine tools of ROA and GT in a merged approach called Option Games (OG) to valuate redevelopment investment projects . Specifically, we embed a Stackelberg competition game in a compound ROA able to capture the sequential logic of redevelopment projects. Although previous studies faced this methodology (see [2]), this paper contributes to the exisisting literature by fitting the OG methodology with the characteristics of redevelopment investment by using discrete time period. Finally, we provide a numerical example to implement the theoretical approach.

Keywords
Game theory ; Options Game ; Public-Private Partnership project ; Real Options Approach ; Urban redevelopment projects

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Bayesian Dynamic Updating in Dynamic Game Model

Jiangjing Zhou
Faculty of Applied Mathematics and Control Processes,
St.Petersburg State University,
Russia
st092028@student.spbu.ru

Ovanes Petrosian
School of Automation, Qingdao University, Qingdao,
China
St. Petersburg State University, Peterhof,
Russia
petrosian.ovanes@yandex.ru

Hongwei Gao
School of Mathematics and Statistics,
Qingdao University,
China
gaohongwei@qdu.edu.cn

Extended abstract [1]

This paper is devoted to examining a dynamic pollution control game model with uncertainty and dynamic updating, where each country's emitted pollution depends on its belief with respect to the unknown parameter and pollution accumulation. The dynamic updating approach is applied to the game model with uncertainty in order to construct a model, where information about the process updates dynamically. In order to deal with the uncertainty, learning introduces two sources of risk about future payoffs: structural uncertainty and uncertainty due to the anticipation of learning.

[1] Speaker:[Jiangjing Zhou : st092028@student.spbu.ru].

The latter renders control and learning non-separable. Therefore, we introduce the Nash equilibrium strategy profile with Bayesian dynamic updating of the dynamic game model with uncertainty. We also show that the limits of the unknown variance for a finite horizon exist. According to the simulations, the addition of learning to a stochastic environment is shown to have a profound effect on the Nash equilibrium strategy profile and the corresponding trajectory.

Keywords
Dynamic games with Bayesian dynamic updating; Looking Forward Approach; Nash equilibrium with Bayesian dynamic updating; Hamilton-Jacobi-Bellman equation; Uncertainty and Learning.

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Social Norms for the stability of International Enviromental Agreements

Marta Biancardi
Department of Economics and Finance,
University of Bari,
marta.biancardi@uniba.it

Lucia Maddalena
Department of Economics, Management and Territory,
University of Foggia,
lucia.maddalena@unifg.it

Giovanni Villani
Department of Economics and Finance,
University of Bari,
giovanni.villani@uniba.it

Extended abstract [1]

This paper is devoted to study the stability of international environmental agreements (IEAs) in a pollution abatement context. Countries can decide to cooperate or to defect. Defector countries determine their abatement levels by minimizing their own total cost whereas, signatory countries decide on their abatement levels by minimizing the aggregate of all cooperators. The game is played in two stage-game; in the first stage countries decide whether or not to become members of an IEA and in the second stage they decide on their abatement levels.
In the model, all countries are affected by the same environmental damage. We also consider an extra cost due to the presence of social norms that are intended to punish countries who behave as defectors. As in [2] and [3], we assume that cooperators are entrusted to punish defectors by applying sanctions. This may be done directly or by alerting authorities. Sanctions

[1] Speaker:[Marta Biancardi, mail address: marta.biancardi@uniba.it].

can be interpreted as trade restrictions or import tariffs, as a measure to encourage cooperation.
Such punishment is costly for defectors, however, in general, this kind of punishment has a negative impact on signatory countries too, in fact, we presume a cost for punishing, this cost is proportional to the punishment.
We assume two scenarios: in the first one, the punishment imposed by cooperators is independent to level of pollution but it is only proportional to the number of defectors while, in the second one, the punishment is directly proportional to the level of pollution, to reflect that the signatories' environmental concern increases with the pollution stock. We model a differential game in order to determine both the optimal path of the abatement levels and stock pollutant as results of feedback Nash equilibria. Stability conditions, such as internal and external stability, are applied showing that different answers about the size of a stable IEA can be obtained respect to environmental damage and punishment.

Keywords

IEA; Non-cooperative game; Social Norms; Feedback Nash Equilibrium; Numerical Applications.

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A differential game model for an efficient pro-vax advertising campaign

Alessandra Buratto
Department of Mathematics Tullio Levi-Civita,
University of Padova, Italy
buratto@math.unipd.it

Rudy Cesaretto
Department of Mathematics Tullio Levi-Civita,
University of Padova, Italy
rudy.cesaretto@unipd.it

Maddalena Muttoni
Department of Mathematics Tullio Levi-Civita,
University of Padova, Italy
maddalena.muttoni@phd.unipd.it

Extended abstract [1]

The discovery of a vaccine for a given disease is hailed as a milestone in medicine since it permits to save lots of lives. In recent times, anti-vaccination movements have been spreading the idea that vaccines can be ineffective and even dangerous. This averse word-of-mouth information has proven to be fairly popular in this contingent period, as declared also in [3], and it represents an obstacle for the diffusion of several vaccinations, such as those against measles, mumps, rubella, meningitis, and papilloma virus.

Due to this phenomenon, the scientific community cannot exclusively concentrate on finding the most effective and efficient medical therapy, it must also plan an efficient pro-vax advertising campaign to instruct people on the importance of being vaccinated.

In [4] and [5] the advertising plan for the vaccination campaign is computed through an optimal control problem, where a policy maker minimizes

[1] Speaker:Alessandra Buratto, buratto@math.unipd.it

the care cost induced by the unvaccinated people and the cost due to the information campaign.

Here, we use the game theory framework to consider the simultaneous communication efforts made by the health-care system (which aims to minimize the costs due to hospitalization and advertising) and a pharmaceutical firm (which produces and sells a given type of vaccine and wants to minimize its loss of earnings and its communication costs). We assume that the number of unvaccinated people increases by the effect of the word-of-mouth publicity as in [2], while it can be reduced thanks to the advertising pro-vax campaign of both players.

A first game approach to this problem appears in [1], where an open-loop Nash equilibrium is computed in a finite time horizon. In order to make a long-term plan of the pro-vax campaign, we consider an autonomous differential game in an infinite time horizon and compute the feedback advertising strategies that constitute its Markovian Nash equilibrium.

Keywords
Dynamic game, Nash equilibrium; Advertising; Vaccination.

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Session: Dynamic Games
Organizers: Gianfranco Gambarelli, Alessandra Buratto

Minimal winning coalitions and orders of criticality

Michele Aleandri
Department of Economics and Finance,
Luiss University, Viale Romania, 32, 00197 Roma, Italia,
maleandri@luiss.it

Marco Dall'Aglio
Department of Economics and Finance,
Luiss University, Viale Romania, 32, 00197 Roma, Italia.
mdallaglio@luiss.it

Vito Fragnelli
Department of of Sciences and Innovative Technologies (DISIT),
University of Eastern Piedmont, Viale T. Michel 11, 15121, Alessandria, Italy.
vito.fragnelli@uniupo.it

Stefano Moretti
Division of LAMSADE, CNRS, Université Paris-Dauphine,
University PSL, 75016, Paris, France.
stefano.moretti@dauphine.fr

Extended abstract [1]

This paper deals with the concept of criticality for players in simple games, originally introduced in [2]. We propose a direct criterion to find the order of criticality of any player in a simple game, based on the cardinality of the minimal blocking coalitions, i.e those coalitions whose complements lose [3, 1]. It turns out that the minimal blocking coalitions coincide with the minimal hitting sets for the family of minimal winning coalitions, i.e. those sets that contain at least one agent for each minimal winning coalition. Moreover, we illustrate the equivalence between the notion of blocking coalitions (or hitting sets) of a simple game and winning coalitions of the

[1] Speaker:[Michele Aleandri - malendri@luiss.it].

dual game. Therefore, we can adapt an algorithm from the rather rich literature on minimal hitting sets generation (see [4]) to compute the order of criticality of players. Therefore, in this paper, we also investigate the use of the notion of orders of criticality (for the grand coalition) to rank players according to their “power” to block decisions in a simple game. To be more specific, we propose four properties that a solution (i.e., a map that associates to any simple game a ranking over the individual players) should satisfy. We then show that a solution satisfies these four properties if and only if it is the criticality-based ranking, which ranks players according to a lexicographic comparison of vectors whose components represent the number of times each player is critical of any order among the minimal winning coalitions.

Keywords
Order of criticality; hitting set; dual game; axiomatic approach.

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SPECIAL SESSION: Cooperatives Games
Session code: CG
Organizers: Gianfranco Gambarelli - Michela Chessa

Dynamic Shapley value for 2-Stage cost-sharing game with arborescences

Peichen Ye
Faculty of Applied Mathematics and Control Processes,
St.Petersburg State University,
Yepeichen@outlook.com

Yin Li
Faculty of Applied Mathematics and Control Processes,
St.Petersburg State University,
liyinrus@outlook.com

Ovanes Petrosian
Faculty of Applied Mathematics and Control Processes,
St.Petersburg State University,
petrosian.ovanes@yandex.ru

Extended abstract [1]

This paper discusses the dynamic Shapley value in cost-sharing game with directed network by establishing minimum cost arborescences. The cooperative behavior of players is defined, and a two-stage network game is considered. At each stage, players form a directed network by adopting strategies and determine a minimum arborescence in the directed network. After the first stage, a special player among the players will drop out the game with a probability, which depends on all actions of players in the first stage. The characteristic function along the cooperative trajectory is defined, the dynamic Shapley value in the game is constructed.

Keywords
Directed Network; Cost-sharing Game; Minimum Cost Arborescences; Dynamic Shapley Value.

[1] Speaker: Peichen Ye, Yepeichen@outlook.com.

Demand commitment bargaining and bounded rationality: An experimental analysis

Michela Chessa
Université Côte d'Azur, CNRS, GREDEG,
France,
michela.chessa@univ-cotedazur.fr

Nobuyuki Hanaki
Institute of Social and Economic Research,
Osaka University, Japan,
nobuyuki.hanaki@iser.osaka-u.ac.jp

Aymeric Lardon
GATE Lyon Saint-Etienne, UMR 5824 CNRS,
Université de Lyon. France,
aymeric.lardon@univ-st-etienne.fr

Takashi Yamada
Yamaguchi University, Japan
tyamada@yamaguchi-u.ac.jp

Extended abstract [1]

The original aim of the *Nash program* [2] was to provide a noncooperative foundation of cooperative solution concepts. Nash started such program designing a noncooperative game which sustained as equilibrium the Nash solution of his cooperative *bargaining problem* [1]. Since then, the Nash program had a very long history and it kept on growing thanks to many theoretical and experimental contributions (the reader is referred to [3] for an exhaustive literature review).

In this paper we experimentally compare three implementations of *Winter demand commitment bargaining mechanism* [4]: a one-period implementation, a two-period implementation with low and with high delay costs.

[1] Speaker:[Michela Chessa, michela.chessa@univ-cotedazur.fr].

Our results show that the three different implementations result in similar outcomes in all our domains of investigation, namely: coalition formation, alignment with the *ex ante* and the *ex post* theoretical prediction and axioms satisfaction. Our result suggests that a simpler implementation that is easier for the players to understand is sufficient in providing accurate results, because the cost of additional complexity introduced by a refinement of the mechanism offsets its benefit.

Keywords
Nash Program; Shapley value; Experiments; Winter mechanism; Coalition formation.

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